SC 617 : Adaptive Control Theory
Project Report

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1 Introduction

This project focuses on addressing the problem of accommodating unknown constant sensor bias in the control of uncertain models. Control of uncertain systems in the presence of actuator and sensor faults is an important and challenging problem. Two traditional approaches to this problem are fault detection, identification (FDI)-based control reconfiguration, and direct adaptive control. We have considered both this approaches in this project that is

- General linear system with model reference adaptive control (MRAC) for state tracking with the following 4 different cases [1]
  - Standard MRAC with biased measurements
  - Feedback-based bias estimation with MRAC
  - Use of asymptotic bias estimator with MRAC
  - Use of non-model-based observers with MRAC
- Sensor fault detection scheme for a class of Lipschitz nonlinear systems with nonlinear and unstructured modeling uncertainty [2]

2 Accommodating Sensor Bias in MRAC for state tracking

In this section we present a brief review and results obtained by implementing [1]. Sensor faults can occur during operation, and if the biased state measurements are directly used with a standard MRAC control law, neither closed-loop signal boundedness, nor asymptotic tracking can be guaranteed and the resulting tracking errors may be unbounded or very large. A modified MRAC law is proposed, which combines a bias estimator with control gain adaptation. It is shown that signal boundedness can be accomplished, although the tracking error may not go to zero. Further if an asymptotically stable sensor bias estimator is available, an MRAC control law is proposed to accomplish asymptotic tracking and signal boundedness. In this paper, it is assumed that all the a priori bias has been compensated for by off-line calibration methods, and trim values of the system states are available at the start of the control operation. Then, direct MRAC is considered in the presence of any constant unknown sensor bias that may appear during the operation. If biased sensor signals are used directly in the MRAC law, signal boundedness and tracking cannot be proved.

2.1 Sensor Bias and Standard MRAC

A Linear time invariant plant is considered:

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = x(t) + \beta$$

where $A \in R^{n\times n}$ is the system matrix assumed to be unknown, $B \in R^{n\times m}$ is a known input matrix, $x(t)$ is the system state, $u(t) \in R^m$ is the control input, $y(t) \in R^n$ is the measured state with an unknown constant bias $\beta \in R^n$.

Objective : Design an adaptive feedback control law using the available measurement $y(t)$ with unknown bias β, such that closed-loop signal boundedness is ensured and the system state $x(t)$ tracks the state of a reference model described by

$$\dot{x}_m(t) = A_m x_m(t) + B_m r(t)$$

It is assumed that the system (A, B) and the reference model (A_m, B_m) satisfy the SFST matching conditions, i.e. there exist gain $K_1 \in R^{n\times m}, K_2 \in R^{m\times m_r}$ such that

$$A_m = A + BK_1{}^T; B_m = BK_2$$

The reference model is designed to capture the desired closed-loop response of the plant. For the simulations that we carried out we designed it by using optimal and robust control method LQR. For the adaptive control scheme, only A_m and B need to be known. Moreover, there exist positive definite matrices $P = P^T, \quad Q = Q^T \in R^{n\times n}$, such that the following Lyapunov inequality is satisfied:

$$A_m^T P + PA < -Q \tag{1}$$

The standard MRAC SFST control law is given by

$$u = \hat{K}_1^T y + \hat{K}_2 r \tag{2}$$

Substituting for $u(t)$ in the system, closed loop system is given by

$$\begin{aligned} \dot{x} &= Ax + B(\hat{K}_1^T y + \hat{K}_2 r) \\ &= (A + Bk_1^T)x + B(\tilde{K}_1^T + \tilde{K}_2 r) + BK_2 r + BK_1^T\beta \end{aligned} \tag{3}$$

where $\tilde{K}_1 = \hat{K}_1 - K_1$; $\tilde{K}_2 = \hat{K}_2 - K_2$. Using the matching conditions we get

$$\dot{x}_m = A_m x + B_m r + B(\tilde{K}_1^T + \tilde{K}_2 r) + BK_1^T \beta \tag{4}$$

Defining the tracking error $e = x - x_m$ and using 4 we get

$$\dot{e} = A_m e + B(\tilde{K}_1^T y + \tilde{K}_2 r) + B{K_1}^T \beta \tag{5}$$

Here, due to bias $x(t)$ cannot be measured and hence $e(t)$ is not available. To achieve signal boundedness, including boundedness of the tracking error, compensation for the sensor bias needs to be implemented.

2.2 Modified MRAC with Sensor Bias Estimation

Let $\hat{\beta}(t)$ denote an estimate of β. Define;

$$\bar{x} = y - \hat{\beta} \tag{6}$$

$$= x + \beta - \hat{\beta} = x + \tilde{\beta} \tag{7}$$

Design an adaptive control law as:

$$u = \hat{K}_1^T y + \hat{K}_2 r + \hat{k}_3 \tag{8}$$

Then, the closed-loop corrected-state equation is

$$\begin{aligned} \dot{\bar{x}} &= Ax + B(\hat{K}_1^T y + \hat{K}_2 r + \hat{k}_3) + \dot{\tilde{\beta}} \\ &= (A + BK_1^T)x + B(\tilde{K}_1^T y + \tilde{K}_2 r + \tilde{k}_3) + BK_2 r + BK_1^T\beta + Bk_3 + \dot{\tilde{\beta}} \end{aligned} \tag{9}$$

Let one more matching condition be $BK_1^T\beta = -Bk_3$, then

$$\dot{\bar{x}} = A_m\bar{x} + B_m r + B(\tilde{K}_1^T y + \tilde{K}_2 r + \tilde{k}_3) - A_m\tilde{\beta} + \dot{\tilde{\beta}} \tag{10}$$

Defining error as $\hat{e} = \bar{x} - x_m$ we have

$$\dot{\hat{e}} = A_m\hat{e} + B(\tilde{K}_1^T y + \tilde{K}_2 r + \tilde{k}_3) - A_m\tilde{\beta} + \dot{\tilde{\beta}} \tag{11}$$

By carrying out the Lyapunov analysis by choosing

$$V = \hat{e}^T P\hat{e} + \sum_i^n \tilde{K}_{1i}^T \Gamma_2^{-1}\tilde{K}_{2i} + \tilde{k}_3^T\Gamma_3^{-1}\tilde{k}_3 + \frac{1}{\eta}\tilde{\beta}^T P\tilde{\beta} \tag{12}$$

where the subscript i denotes the i^{th} column of $\tilde{K}_1, \tilde{K}_2$ and differentiating w.r.t time and using properties of matrix trace the gain update laws are given by

$$\begin{aligned} \dot{\hat{K}}_1 &= -\Gamma_1 y\hat{e}^T PB \\ \dot{\hat{K}}_2 &= -\Gamma_2 B^T P\hat{e}r^T \\ \dot{\hat{k}}_3 &= -\Gamma_3 B^T P\hat{e} \end{aligned} \tag{13}$$

where $\Gamma_1 \in R^{n\times n}$, $\Gamma_2 \in R^{m\times m}$ and $\Gamma_3 \in R^{m\times m}$ are constant symmetric positive definite matrices and P was defined in Eq.1 and the bias estimation law is

$$\dot{\hat{\beta}} = -\eta P^{-1} A_m^T P\hat{e} \tag{14}$$

where $\eta \in R$ is a tunable positive constant gain. The adaptive control law in Eq. 13 and 14 guarantees stability (signal boundedness) and bounded tracking error.

3 MRAC with Asymptotic Bias Estimation

Suppose an asymptotically stable bias estimator is available. This section illustrates how such an estimator can be employed in a MRAC setting to achieve asymptotic state tracking. Consider a bias estimator with estimation error dynamics of the form

$$\dot{\tilde{\beta}} = A_\beta \tilde{\beta} \tag{15}$$

$A_\beta \in R^{n \times n}$ is a known asymptotically stable matrix, such that $\lim_{t\to\infty} \tilde{\beta}(t) = 0$. Defining the adaptive control law as in Eq. 8 and proceeding on similar lines Eq. 13 can be obtained. Asymptotic stability can be easily proved by Lyapunov analysis by choosing

$$V = \hat{e}^T P \hat{e} + \sum_i^n \tilde{K}_{1i}^T \Gamma_2^{-1} \tilde{K}_{2i} + \tilde{k}_3^T \Gamma_3^{-1} \tilde{k}_3 + \tilde{\beta}^T P_\beta \tilde{\beta} \tag{16}$$

where $P_\beta = P_\beta^T$ satisfies

$$A_\beta^T P_\beta + P_\beta A_\beta < -Q \tag{17}$$

for some positive definite matrix $Q_\beta = Q_\beta^T$.

4 Non-Model-Based Observers for Longitudinal Aircraft Dynamics

In this section it is illustrated how sensor bias estimators can be developed and used in conjunction with MRAC. The bias estimates also generate estimates of other measurable states. It is shown, how these additional state estimates, and the bias estimates are combined with MRAC to yield asymptotic state tracking even in the presence of unknown sensor bias. We consider a fourth-order longitudinal dynamic model of an aircraft. The states are: true airspeed $v(m/s)$, angle-of-attack $\alpha(deg)$, pitch angle $\theta(deg)$, and pitch rate $q(deg/s)$.

4.1 Rate Gyro Bias Observer

Consider that the available state measurement ($y = x + \beta$) has an unknown constant bias in the pitch rate measurement, such that

$$\beta = [0 \quad 0 \quad 0 \quad 10]^T \tag{18}$$

where b is an unknown constant. Thus, the measured pitch rate ($q_m(t)$) can be written as

$$\begin{aligned} q_m &= q + b = \dot{\theta} + b \\ \dot{\theta} &= -b + q_m \end{aligned} \tag{19}$$

Suppose an independent bias-free measurement $y_0(t)$ of the pitch angle is available as

$$y_0(t) = \theta$$

Since, the unknown bias b is constant, the Eq. 19 can be augmented with $\dot{b} = 0$ to yield:

$$\begin{aligned} \dot{x}_0 &= A_0 x_0 + B_0 q_m \\ y_0 &= C_0 x_0 \end{aligned} \tag{20}$$

where

$$\begin{aligned} x_0 &= [\theta \quad b]^T \\ A_0 &= \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \\ B_0 &= [1 \quad 0]^T \\ C_0 &= [1 \quad 0] \end{aligned} \tag{21}$$

Since (C_0, A_0) is observable, an estimator can be constructed as

$$\dot{\hat{x}} = A_0\hat{x_0} + B_0 q_m + H(y_0 - \hat{y}_0) \tag{22}$$

where $H \in R^2$ is an observer gain, we have used Kalman filter, therefore H is Kalman gain,

$$H = [h_1 \quad h_2]^T$$

Using Eq. 20 and 22 we get the following estimation error dynamics

$$\dot{\tilde{x}}_0 = (A_0 - HC_0)x_0 \tag{23}$$

Since, the actual θ is measurable, the initial estimate of $\hat{\theta}(0)$ can be initialized to its true value, such that $\tilde{\theta} = 0$, while $\hat{b} = 0$ can be chosen arbitrarily. The control law is defined as in Eq.8 such that using the matching conditions the following closed-loop corrected-state equation is obtained:

$$\dot{\bar{x}} = A_m\bar{x} + B_m r + B(\tilde{K}_1^T y + \tilde{K}_2 r + \tilde{k}_3) - A_m\tilde{\beta} + \dot{\tilde{\beta}} \tag{24}$$

Using the above equations the error dynamics are given by

$$\begin{aligned} \dot{\hat{e}} &= A_m\hat{e} + B(\tilde{K}_1^T y + \tilde{K}_2 r + \tilde{k}_3) - A_{m4}\tilde{b} - \bar{h}_2\tilde{\theta} \\ &= A_m\hat{e} + B(\tilde{K}_1^T y + \tilde{K}_2 r + \tilde{k}_3) - M\tilde{x}_0 \end{aligned} \tag{25}$$

where

$$\begin{aligned} \bar{h}_2 &= [0 \quad 0 \quad 0 \quad h_2]^T \\ M &= [\bar{h}_2 \quad A_{m4}] \end{aligned} \tag{26}$$

and A_{m4} is the 4^{th} column of A_m. Signal boundedness and asymptotic tracking using such a bias estimator can be shown by using Lyapunov equation by using

$$V = \hat{e}^T P\hat{e} + \sum_i^n \tilde{K}_{1i}^T\Gamma_2^{-1}\tilde{K}_{2i} + \tilde{k}_3^T\Gamma_3^{-1}\tilde{k}_3 + \tilde{x}_0^T P_0\tilde{x}_0 \tag{27}$$

The adaptive gains are given by Eq. 13.

5 Simulations

Simulations are performed on a fourth-order longitudinal dynamics model of a large transport aircraft in a wings-level cruise condition with known nominal trim conditions. The reference input command superimposed with white noise is shown in 1. The state variables for a fourth-order longitudinal dynamics model are: true airspeed $v(m/s)$, angle-of-attack $\alpha(deg)$, pitch angle $\theta(deg)$, and pitch rate $q(deg/s)$. The actuators are elevator and engine throttle input, which produce control inputs $u_e(deg)$ and u_t, respectively. Actuator dynamics are not considered. The system matrices are given by:

$$\begin{aligned} A &= \begin{bmatrix} -0.0062 & -.0815 & -0.1709 & -0.0026 \\ -0.0344 & -0.5717 & 0 & 1.0050 \\ 0 & 0 & 0 & 1.0 \\ 0.0115 & -1.0490 & 0 & -0.6803 \end{bmatrix} \\ B &= \begin{bmatrix} 0 & 1.3827 \\ -11.4027 & -0.0401 \\ 0 & 0 \\ -44.5192 & 0.8824 \end{bmatrix} \end{aligned} \tag{28}$$

The state is $x(t) = [v \quad \alpha \quad \theta \quad q]^T$, and the control input is $u(t) = [u_e \quad u_t]^T$. K_1 is chosen by LQR and $K_2 = I_2$.

5.1 Case I: Standard MRAC with biased measurements

The unknown constant bias in the state measurement is arbitrarily chosen as

$$\beta = [5 \quad 2 \quad -1 \quad 10]^T \tag{29}$$

Using the biased measurements, the standard MRAC control law of Eq.2 and 13 is implemented with

$$\hat{K}_1(0) = 0.5K_1, \hat{K}_2(0) = 0.5K_2, \Gamma_1 = .005I_4, \Gamma_2 = .005I_2$$

The plant and reference model states, tracking errors, and control input are shown in 2, 3 and 4 respectively. As stated in Section 2.1, the closed-loop signal boundedness or bounded tracking error cannot be proved. However, in this example, the tracking errors appear to approach some non-zero constant values.

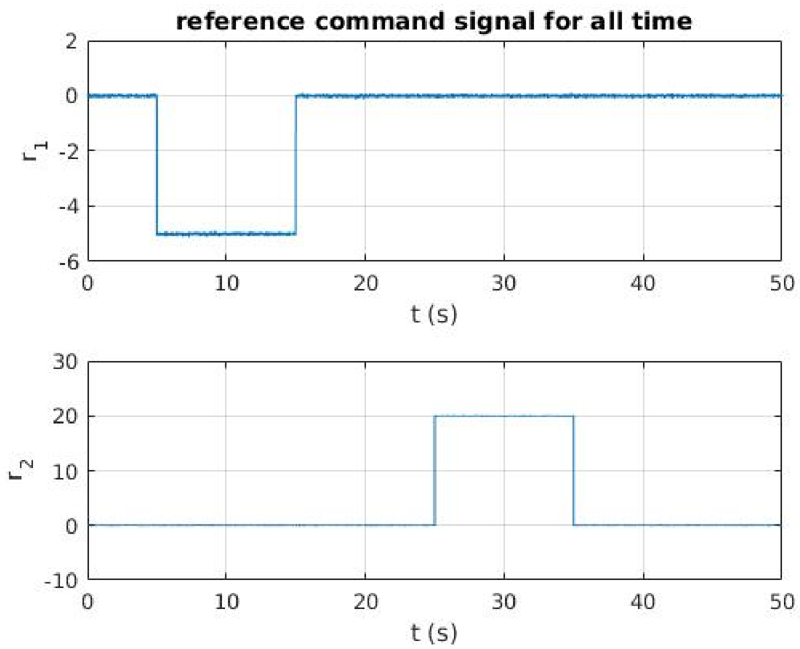

Figure 1: Reference command for all cases with SNR = 30

5.2 Case II: Feedback-based bias estimation with MRAC

With the same value of β as for case I the control law in Eq. 8 is implemented where the bias estimate is generated using the bias estimation law in Eq. 13 and 14. The gain $\hat{k}_3$ is initialized to zero with adaptation rate $\Gamma_3 = .002I_2$ and $\eta = .01$ while other adaptive gains are chosen as in case I. The plant and reference model states, tracking errors, control input, and bias estimates are shown in 5,6, 7 and 8. Except for velocity the controller and the bias estimator effectively compensate for the sensor bias, and achieve an acceptable tracking performance. The bias estimates converge to values that are not necessarily the true values.

5.3 Case III: Use of asymptotic bias estimator with MRAC

For the same sensor bias as above the control law in Eq. 8 is implemented and bias estimate is generated using the bias estimation law in Eq. 15. The adaptive gain initialization and adaptation gains are chosen same as in Cases I and II. The matrix A_β is arbitrarily chosen as $A_\beta = 1.1A_m$. The bias estimate converges to constant values as shown in 12. The plant and reference model states, tracking errors, and control input are shown in 9,10 and 11 respectively. The tracking errors converge to zero asymptotically.

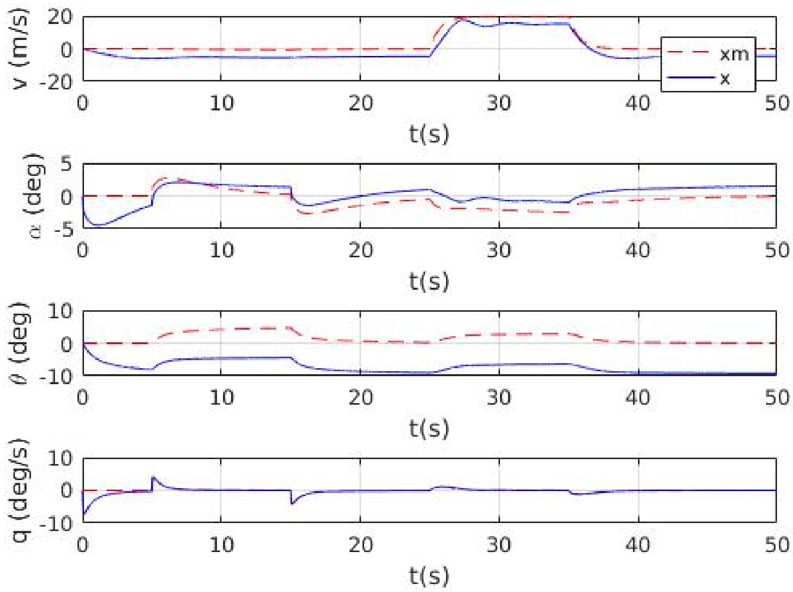

Figure 2: Case 1: Plant and reference model states

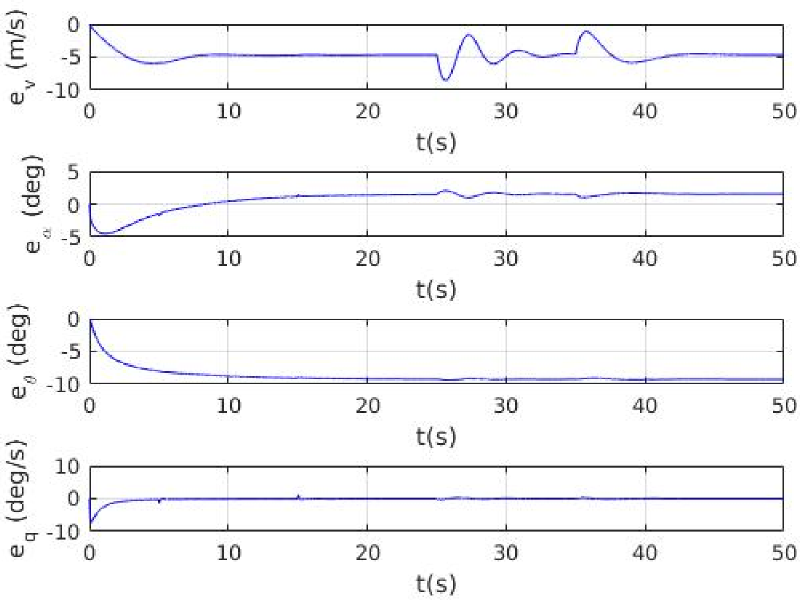

Figure 3: Case 1: State tracking error

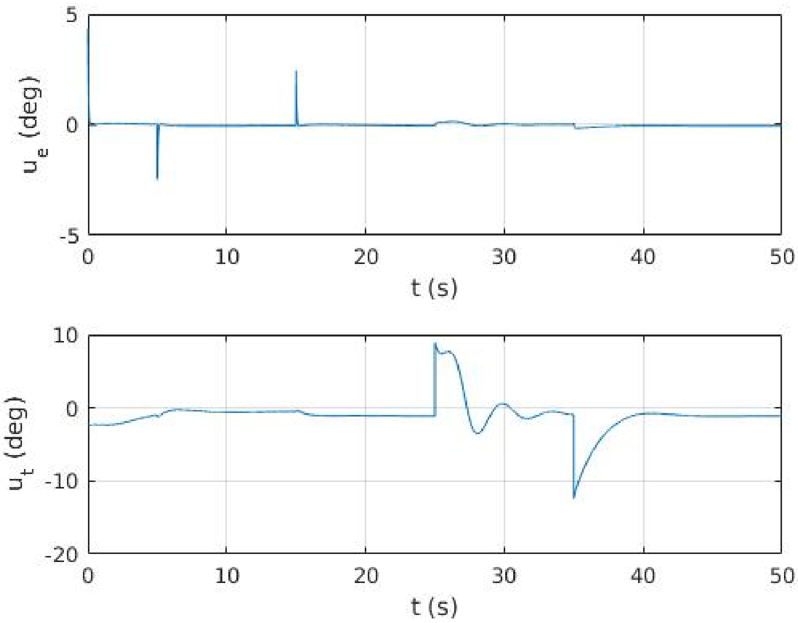

Figure 4: Case 1: Control input

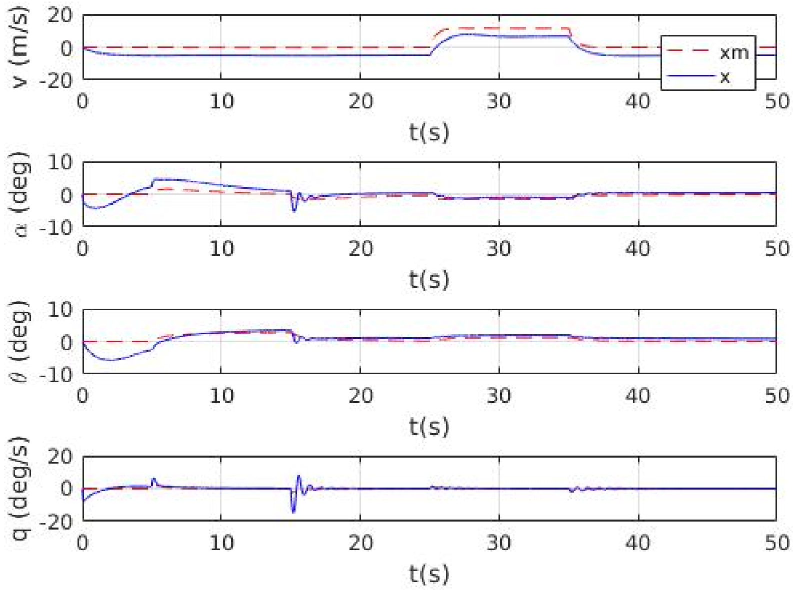

Figure 5: Case II: Plant and reference model states

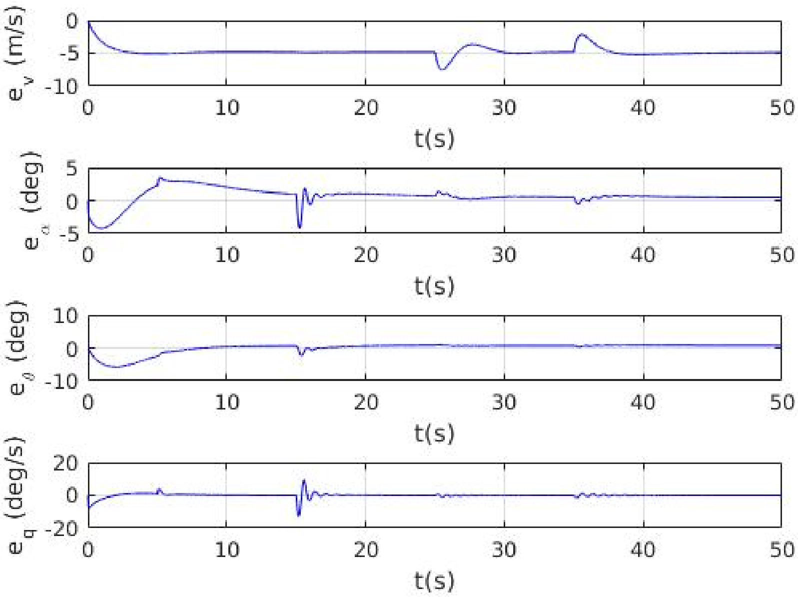

Figure 6: Case II: State tracking error

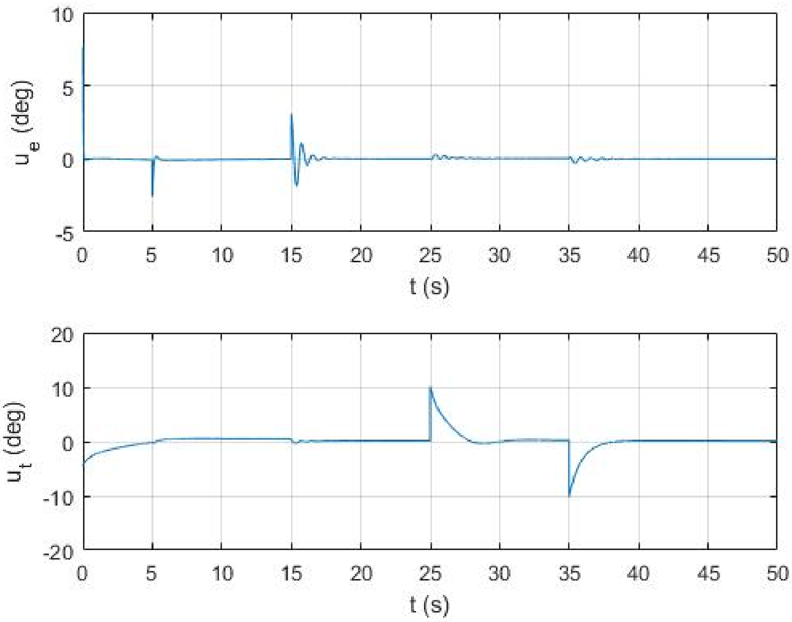

Figure 7: Case II: Control input

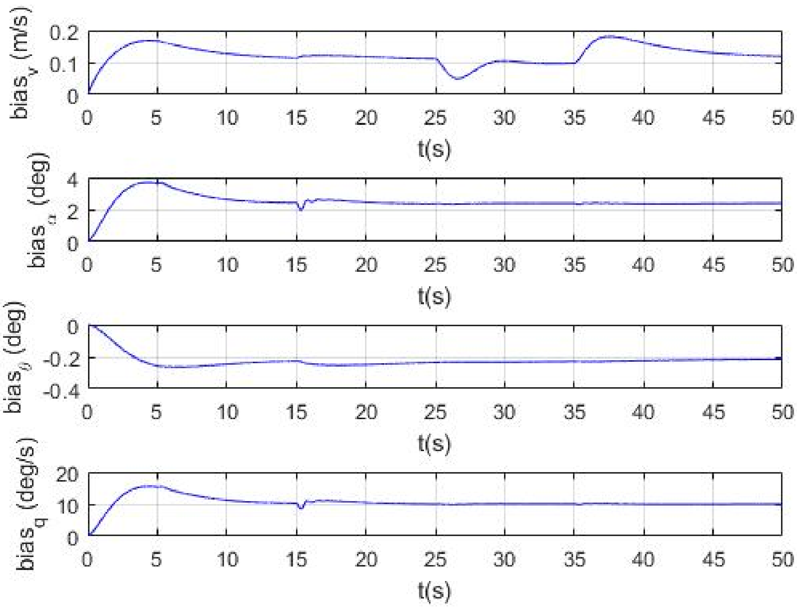

Figure 8: Case II: Bias Estimates

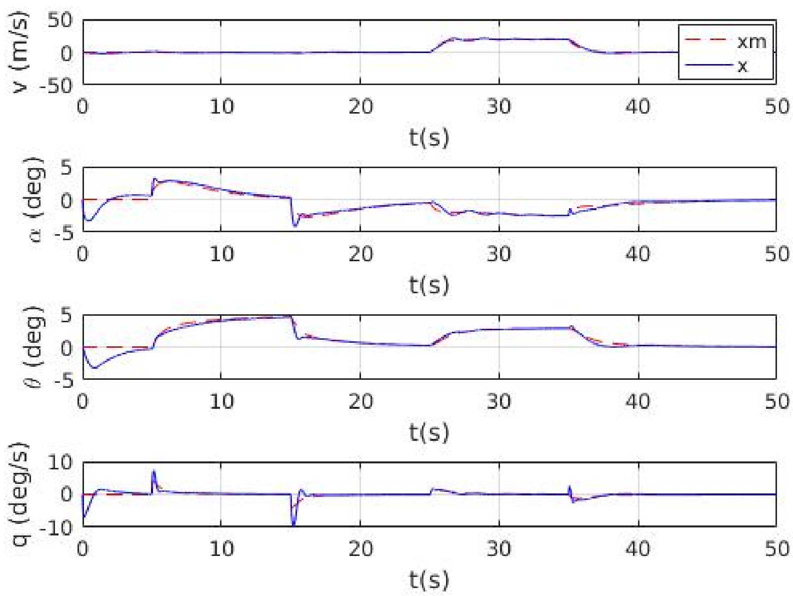

Figure 9: Case III: Plant and reference model states

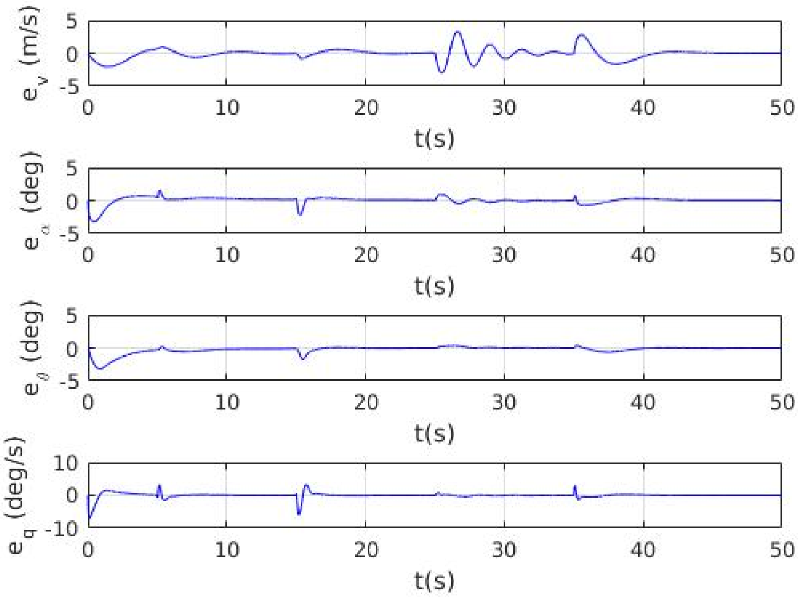

Figure 10: Case III: State tracking error

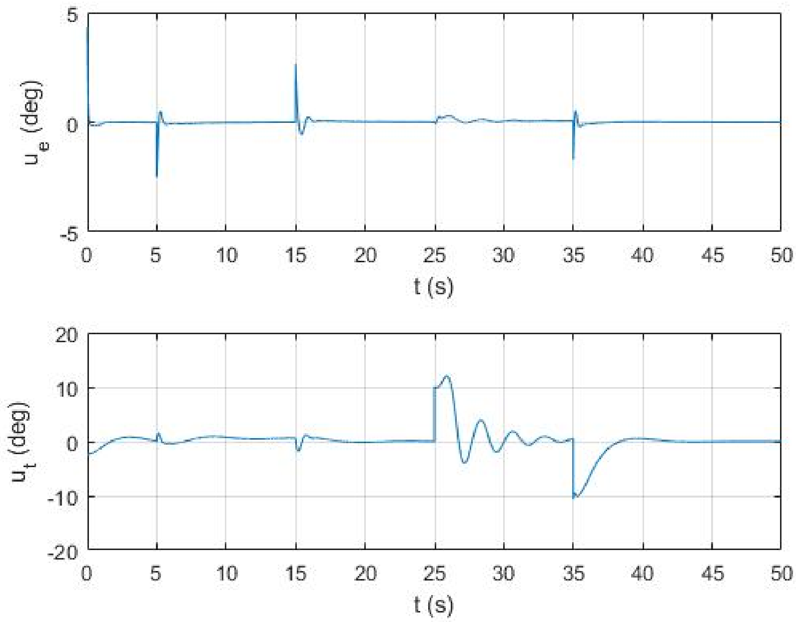

Figure 11: Case III: Control input

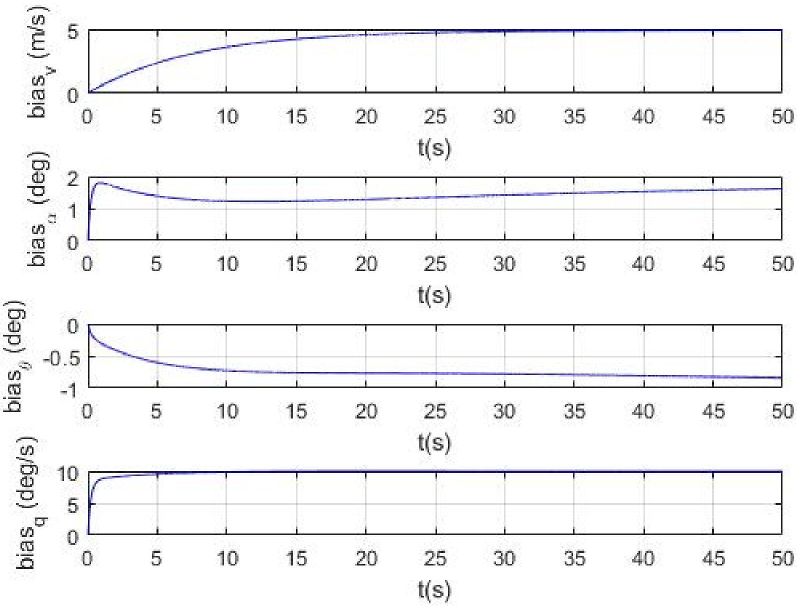

Figure 12: Case III: Bias Estimates

5.4 Case IV: Use of non-model-based observers with MRAC

The bias vector considered in this case is different compared to the other cases because in order to use the non-model-based observer it is assumed that the bias exists only in the pitch-rate measurements, while reliable bias-free measurements of airspeed, angle-of-attack and pitch angle are available and is given by Eq.18. The adaptive gain initialization and adaptation gains are chosen same as in Case II. The bias estimate converges to constant values as shown in 16. The plant and reference model states, tracking errors, and control input are shown in 13,14 and 15 respectively. The tracking errors converge to zero asymptotically.

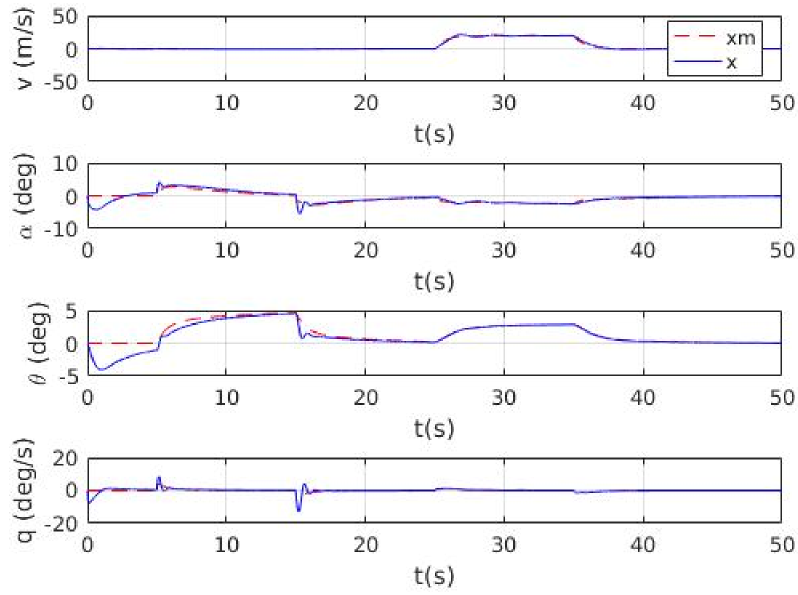

Figure 13: Case IV: Plant and reference model states

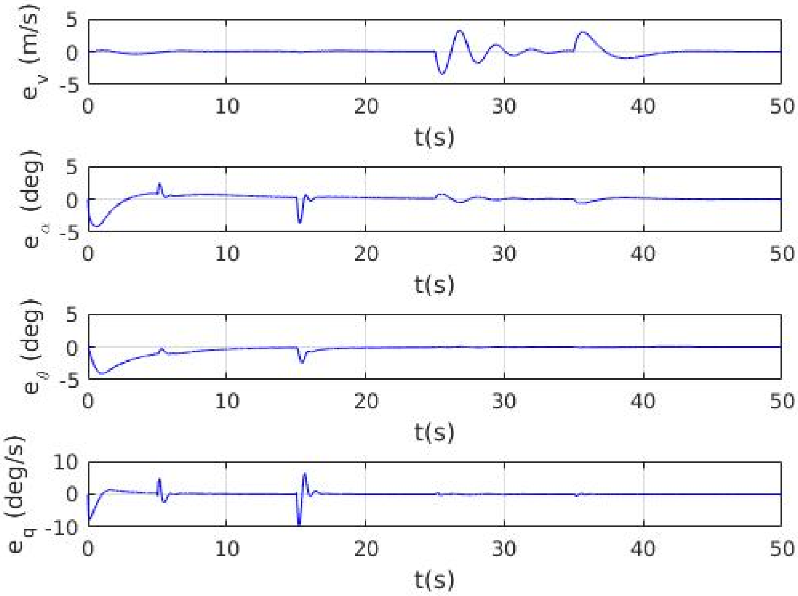

Figure 14: Case IV: State tracking error

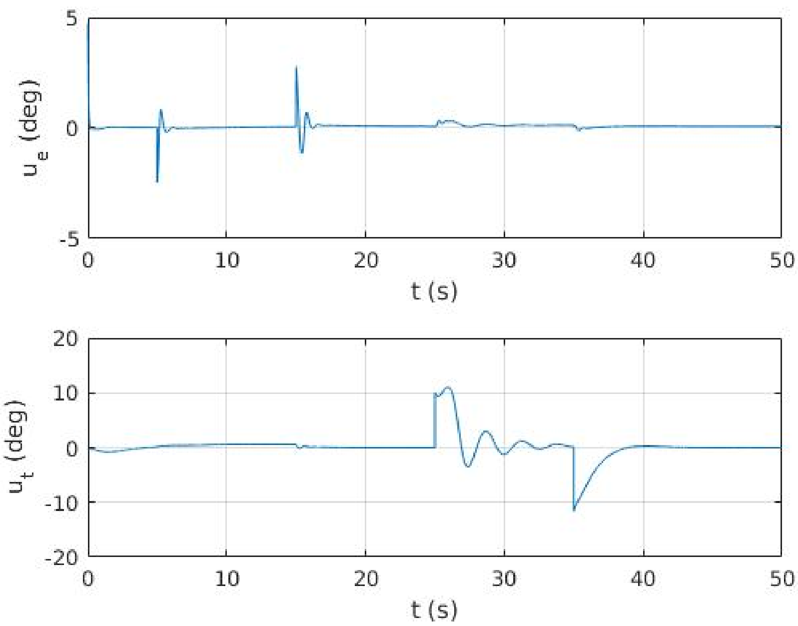

Figure 15: Case IV: Control input

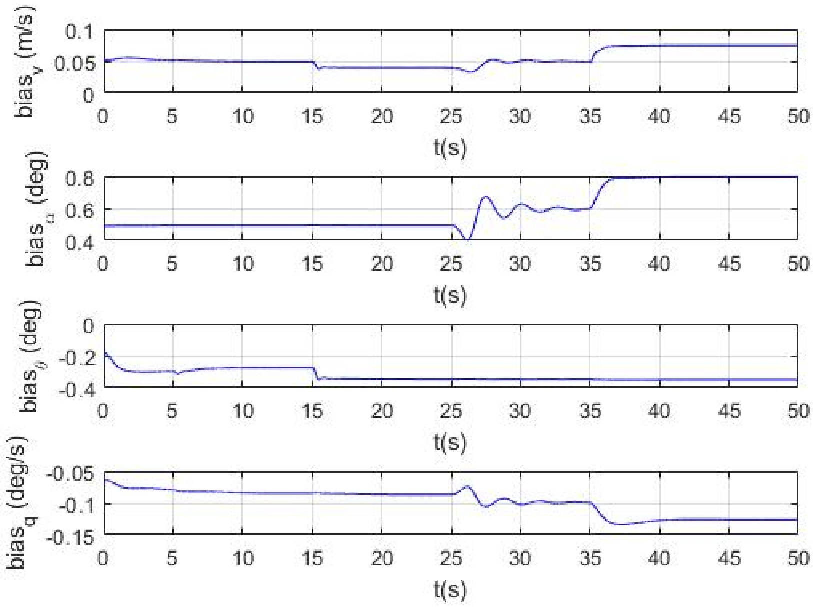

Figure 16: Case IV: Bias Estimates

6 Discussion

The steady-state tracking errors improve as the design progresses from Case I to Case III along with the improvements in the steady-state condition of the airplane. In Case 1, using just the standard MRAC control law, the airplane trims at 5 (m/s) too slow (consistent with the 5 (m/s) too high sensor measurement) and at a flight-path angle ($\gamma = \theta - \alpha$) of about -8 (deg) (descending steeper than a normal landing approach). In Case II, with the addition of feedback-based bias estimation to the MRAC control law, the airplane flight-path angle is partially corrected to about -1 (deg) while the speed error is not affected. In Case III, with an asymptotic bias estimator, both the flight-path angle and the speed are fully corrected. In Case IV, the bias only exists in pitch-rate measurements. The non-model-based observer developed in Section 4 combined with MRAC is able to achieve asymptotic tracking

7 Sensor Bias Fault Diagnosis in a Class of Nonlinear Uncertain Systems with Lipschitz Nonlinearities

This paper by *Xiaodong Zhang* presents a sensor fault detection and isolation scheme for a class of Lipschitz nonlinear systems with nonlinear and unstructured modeling uncertainty. We have studied only the Fault detection scheme for this project.

7.1 Problem Formulation

Consider a class of nonlinear multi-input-multi-output (MIMO) dynamic systems described by

$$\begin{aligned} \dot{x} &= Ax + D\zeta(x,u) + g(y,u) + \psi(x,u,t) \\ y &= Cx + \beta(t-T_0)F\theta(t) \end{aligned} \tag{30}$$

where $\zeta : R^n \times R^m \longmapsto R^v$, $\quad g : R^p \times R^m \longmapsto R^n$. $\quad$ *and* $\quad \psi : R^n \times R^m \times R^+ \longmapsto R^n$.

The constant matrices $D \in R^{n\times v}$ *and* $C \in R^{p\times n}$ with $v \le p$ are of full rank, (A,C) is an observable pair. The model is given by

$$\begin{aligned} \dot{x}_N &= Ax_n + D\zeta(x_N,u) + g(y_N,u) \\ y_N &= Cx_n \end{aligned} \tag{31}$$

ψ represents the modeling uncertainty. The term $F\theta(t)$ represents the time varying bias of the sensor fault, and the function $\beta(t - T_0)$ characterizes the fault time profile, where $T_0 > 0$ is the unknown fault occurrence time. Author assumes the occurrence of a single sensor fault. Hence, the fault distribution vector $F \in R^p$ has only one non-zero entry, which represents the corresponding corrupted output measurement, and the scalar $\theta(t) \in R$ is the magnitude of the time varying sensor bias. Following are few assumptions taking into account for arriving at Fault detection scheme.

Assumption 1

The matrix D in Eq. 30 satisfies:

- rank$(CD) = v$
- all the invariant zeros of (A,D,C) (if any) lie in the left half plane

Then under assumption 1 there exists a linear change of coordinates $z = Tx = [z_1^T \quad z_2^T]^T$ with $z_1 \in R^{(n-p)}$ and $z_2 \in R^p$, such that

- $TAT^{-1} = \begin{bmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} \\ \mathcal{A}_{21} & \mathcal{A}_{22} \end{bmatrix}$ where the matrix $\mathcal{A}_{11} \in R^{(n-p)\times(n-p)}$ is stable
- $TD = [0 \quad D_2]^T$, where $D_2 \in R^{p\times v}$
- $CT^{-1} = [0 \quad I_p]$

In the new coordinate system the system (Eq. 30) can be described by

$$\begin{aligned} \dot{z}_1 &= \mathcal{A}_{11}z_1 + \mathcal{A}_12z_2 + \psi_1(y,u) + \eta_1(z,u,t) \\ \dot{z}_2 &= \mathcal{A}_{21}z_1 + \mathcal{A}_{22}z_2 + D_2\zeta(z,u) + \psi_2(y,u) + \eta_2(z,u,t) \\ y &= z_2 + \beta(t - T_0)F\theta(t) \end{aligned} \tag{32}$$

where

$$\begin{bmatrix} \psi_1(y,u) \\ \psi_2(y,u) \end{bmatrix} = Tg(y,u)$$
$$\begin{bmatrix} \eta_1(z,u,t) \\ \eta_2(z,u,t) \end{bmatrix} = T\psi(T^{-1}z,u,t)$$

With a more general structure of the system nonlinearity, Eq. 32 can be extended to

$$\begin{aligned} \dot{z}_1 &= \mathcal{A}_{11}z_1 + \mathcal{A}_{12}z_2 + \psi_1(y,u) + \eta_1(z,u,t) \\ \dot{z}_2 &= \mathcal{A}_{21}z_1 + \mathcal{A}_{22}z_2 + \rho(z,u) + \psi_2(y,u) + \eta_2(z,u,t) \\ y &= \bar{\mathcal{C}}z_2 + \beta(t - T_0)F\theta(t) \end{aligned} \tag{33}$$

where $\rho : R^n \times R^m \mapsto R^p$ is a smooth vector field, and $\bar{\mathcal{C}} \in R^{p\times p}$ is a nonsingular matrix

Assumption 2

The modeling uncertainties, represented by $\eta_1 \quad and \quad \eta_2$ in Eq. 33, are unstructured and unknown nonlinear functions of z, u, t but bounded by given functionals,i.e. $\forall(z,y,u) \in \mathcal{Z} \times \mathcal{Y} \times \mathcal{U}, \forall t > 0$

$$\begin{aligned} |\eta_1(z,u,t)| &\le \bar{\eta}_1(y,u,t) \\ |\eta_2(z,u,t)| &\le \bar{\eta}_2(y,u,t) \end{aligned} \tag{34}$$

where the bounding functions $\bar{\eta}_1(y,u,t)$ and $\bar{\eta}_2(y,u,t)$ are known and uniformly bounded in $\mathcal{Y} \times \mathcal{U} \times \mathcal{R}^+$. Here, $\mathcal{Z} \subset R^n, \mathcal{U} \subset R^m \quad and \quad \mathcal{Y} \subset R^p$ are compact sets of admissible states, inputs, and outputs, respectively. This assumption characterizes the class of modeling uncertainties under consideration. The bounds on the unstructured modeling uncertainties are needed in order to be able to distinguish between the effects of faults and modeling uncertainty.

Assumption 3

The state z remains bounded before and after fault occurrence, i.e.,$z(t) \in L_\infty, \forall t \geq 0$.

This assumption assumes that the feedback control system is capable of retaining the boundedness of the state variables before and after the occurrence of a fault. This is a technical assumption required for well posedness since the FDI design that we consider does not influence the closed-loop dynamics and stability.

Assumption 4

The known nonlinear term $\rho(z,u)$ in Eq. 33 is uniformly Lipschitz in $u \subset U$, i.e., $\forall z, \hat{z} \in \mathcal{Z}$,

$$|\rho(z,u) - \rho(\hat{z},u)| \leq \gamma|z - \hat{z}| \tag{35}$$

where γ is the known Lipschitz constants.

In this assumption a known bound on the rate of change of $\theta(t)$ is assumed. In practice, the bound α can be set by exploiting some a priori knowledge on the sensor bias developing dynamics. In the case of a constant sensor bias, we simply set $\alpha = 0$.

7.2 Fault Detection Method

The sensor fault detection and isolation (FDI) architecture is based on a set of p + 1 nonlinear adaptive estimators, where p is the number of sensors under monitoring. One of the nonlinear estimators is the *fault detection estimator* (FDE) used for detecting the occurrence of faults, while the remaining p nonlinear adaptive estimators are *fault isolation estimators* (FIEs), which are activated after fault detection to determine the particular faulty sensor.

Fault Detection Scheme

Based on the model given in Eq (32) , the FDE is represented as follows:

$$\dot{\hat{z}}_1 = \mathcal{A}_{11}\hat{z}_1 + \mathcal{A}_{12}\bar{\mathcal{C}}^{-1}y + \psi_1(y,u) \tag{36}$$

$$\dot{\hat{z}}_2 = \mathcal{A}_{21}\hat{z}_1 + \mathcal{A}_{22}\hat{z}_2 + \rho(\hat{z},u) + \psi_2(y,u) + L(y - \hat{y}) \tag{37}$$

$$\hat{y} = \bar{\mathcal{C}}\hat{z}_2 \tag{38}$$

where $\hat{z}_1$, $\hat{z}_2$, and $\hat{y}$ denote the estimated state and output variables, respectively, $L \in \Re^{p\times p}$ is a design gain matrix, and $\hat{z} \equiv [(\hat{z}_1)^T(\bar{\mathcal{C}}^{-1}y)^T]^T$. The initial conditions are $\hat{z}_1(0) = 0$ and $\hat{z}_2(0) = \bar{\mathcal{C}}^{-1}y(0)$. Let $\tilde{z}_1 \equiv z_1 - \hat{z}_1$ and $\tilde{z}_2 \equiv z_2 - \hat{z}_2$ denote the state estimation errors, and $\tilde{y} \equiv y - \hat{y}$ denote the output estimation error. Then, before fault occurrence (i.e., for $t < T_0$), we have

$$\dot{\tilde{z}}_1 = \mathcal{A}_{11}\tilde{z}_1 + \eta_1(z,u,t) \tag{39}$$

$$\dot{\tilde{z}}_2 = \mathcal{A}_{22}\tilde{z}_2 + \mathcal{A}_{21}\tilde{z}_1 + \rho(\hat{z},u) - \rho(\hat{z},u)\eta_2 \tag{40}$$

$$\tilde{y} = \bar{\mathcal{C}}\tilde{z}_2 \tag{41}$$

where $\bar{\mathcal{A}}_{22} \equiv \mathcal{A}_{22} - L\bar{\mathcal{C}}$ Note that , since $\bar{\mathcal{C}}$ is no singular we can always choose L to make $\bar{\mathcal{A}}_{22}$ stable. the following equation is obtained through property (4) and using triangle law of inequality stated in [2].

$$|\tilde{z}_1(t)| \leq k_0 w_0 e^{-\lambda_0 t} + k_0 \int_0^t e^{-\lambda_0(t-\tau)}\bar{\eta}_1(y,u,\tau)d\tau \tag{42}$$

where k_0 and λ_0 are positive constants that satisfy $\|e^{\mathcal{A}_{11}t}\| \leq k_0 e^{-\lambda_0 t}$, and w_0 is such that $|\tilde{z}_1(0)| \leq w_0$.

Similarly, using equations (39) and (34) , each component of output estimation error , i.e., $\tilde{y}_j(t) = \bar{\mathcal{C}}_j\tilde{z}_2(t)$, j = 1, ..., p, is given by

$$\tilde{y}_j(t) = \int_0^t \bar{\mathcal{C}}_j e^{\bar{\mathcal{A}}_{22}(t-\tau)}[\mathcal{A}_{21}\tilde{z}_1(\tau) + \rho(z,u) - \rho(\hat{z},u0)]d\tau + \int_0^t \bar{\mathcal{C}}_j e^{\bar{\mathcal{A}}_{22}(t-\tau)}\eta_2(z,u,\tau)d\tau \tag{43}$$

Since before fault occurrence (i.e., for $t < T_0$, we know that

$$z(\tau) - \hat{z}(\tau) = [\tilde{z}_1 \quad 0]^T \tag{44}$$

Therefore, we have $|z(\tau) - \hat{z}(\tau)| = |\tilde{z}_1|$, By using the properties (4), (5), and (11) stated in [2], we obtain

$$|\tilde{y}_j(t)| = k_j \int_0^t e^{-\lambda_j(t-\tau)}[(\|\mathcal{A}_{21}\| + \gamma)|\tilde{z}_1(\tau)|]d\tau + k_j \int_0^t e^{-\lambda_j(t-\tau)}\bar{\eta}_2(y, u, \tau)d\tau \tag{45}$$

where k_j and λ_j are positive constants that satisfy $|\bar{\mathcal{C}}_j e^{\bar{\mathcal{A}}_{22}t} \leq k_j e^{-\lambda_j t}$. By using (10) and (12) sited in [], we obtain $|\tilde{y}_j(t)| \leq \nu_j(t)$, where

$$\nu_j(t) \triangleq k_j \int_0^t e^{-\lambda_j(t-\tau)}[(\|\mathcal{A}_{21}\| + \gamma)\chi(\tau)]d\tau + k_j \int_0^t e^{-\lambda_j(t-\tau)}\bar{\eta}_2(y, u, \tau)d\tau \tag{46}$$

with $\chi(\tau) \triangleq k_0 w_0 e^{-\lambda_0 t} + k_0 \int^t{}_0 e^{-\lambda_0(t-\tau)}\bar{\eta}_2(y, u, \tau)d\tau$

Accordingly, we have the following:

Result

The decision on the occurrence of a fault (detection) is made when the modulus of at least one component of the output estimation error (i.e., $\tilde{y}_j(t)$) exceeds its corresponding threshold $\nu_j(t)$ given by Eq. 46. The fault detection time T_d is defined as the first time instant such that $|\tilde{y}_j(T_d)| > \nu_j(T_d)$, for some $T_d \geq T_0$ and some $j \in 1, \cdots$, p, that is,

$$T_d \triangleq inf \bigcup_{j=1}^{p} t \geq 0 : |\tilde{y}_j(t) > \nu_j(t)|$$

This guarantees that there will be no false alarms before fault occurrence (i.e., for $t \leq T_0$)

References

[1] Accommodating Sensor Bias in MRAC for state tracking. *Parag Patre* and *Suresh M. Joshi*

[2] Sensor Bias Fault Diagnosis in a Class of Nonlinear Uncertain Systems with Lipschitz Nonlinearities. *Xiaodong Zhang*. 2010 American Control Conference Marriott Waterfront, Baltimore, MD, USA June 30-July 02, 2010