Relative Autonomous Navigation Without Communication between Spacecrafts Using Line of Sight Measurements

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***Abstract*— A novel algorithm to use line of sight (LOS) measurements for relative position, attitude and angular rate estimation for autonomous spacecraft navigation is developed. Traditional relative attitude navigation is based on gyro measurements from the spacecrafts in formation for estimating angular rates. But it requires information exchange between the two spacecrafts and continuous availability of gyro data. The loss of gyro data can result in high propagation errors. The approach presented here can determine relative angular velocity in the event of gyro failures or communication delays. Previously, an algorithm for spacecraft angular rate estimation for star tracker based attitude determination had been proposed. In this paper, this algorithm is extended to estimate relative angular rates from the LOS measurements without assuming any on-board star tracker or gyros. An extended Kalman filter (EKF) is used here to estimate the relative motion. The state of the EKF consists of relative quaternion, angular velocity, position and velocity. The dynamic model of the relative motion is based on generalized Clohessy and Wiltshire equations. The angular acceleration of the follower spacecraft is modeled by Gaussian white noise. This is done for estimating relative angular rates. Numerical simulations are carried out to analyze the performance of this algorithm.**

I. INTRODUCTION

Spacecraft formation flying is an important technology for modern day space agencies, with application to areas like stereographic imaging, synthetic apertures and autonomous orbital rendezvous. They require that relative attitude and position between spacecrafts is maintained. Autonomous proximity operations are required for International space station repair, refueling and servicing. In a leader follower configuration, the spacecraft about which all the other spacecrafts are orbiting is refereed to as leader and the remaining spacecrafts as followers. In the past decade, many methods to obtain LOS vectors have been proposed. Demonstration of Autonomous Rendezvous Technology (DART) and Orbital Express made use of Advanced Video Guidance Sensor (AVGS) to obtain line of sight measurements. The AVGS computes and reports a 6-DOF-vector for the leader coordinate system relative to the AVGS coordinate system. This vector consists of range, azimuth and elevation of the leader frame [1]. Many research studies also make use of GPS (Global Positioning System)-like technology to obtain relative attitude and position, but this limits the use to low earth orbit operations only. Kim et al. discuss a vision based navigation (VISNAV) to obtain LOS vectors, which comprises of an optical sensor [2]. Several light emitting diodes, called beacons, are fixed on the leader frame (spacecraft) and an optical sensor on the follower frame (spacecraft). This optical sensor works analogously to radar to determine relative range and attitude.

Factors like reliability, accuracy, and cost of the sensor determine its suitability for a specific space mission. We have considered VISNAV algorithm as discussed in [2].The main objective of this paper is to present an extended Kalman filter (EKF) formulation for relative spacecraft navigation using only LOS measurements. Generally, three axis gyros are used on board for body angular rate information. Autonomous proximity operations between spacecrafts then requires information exchange of their respective body angular rates. A gyro failure or poor communication between the spacecrafts could lead to the failure of entire mission. P. Singla et al. have developed an efficient algorithm for estimation of spacecraft body angular rates in the absence of gyro rate data for a start tracker mission [3]. We have extended this algorithm to obtain relative angular velocity between the spacecrafts. This eliminates the need to know individual spacecraft's angular velocity.

This paper is organized as follows. First, a brief review of dynamical model for relative translational motion is given followed by quaternion based attitude kinematics model and equations for relative rotational motion. Then the basic equations for VISNAV system are shown. Next, an algorithm for estimating relative angular velocity using Kalman filtering is discussed. Subsequently, an EKF formulation to estimate relative attitude, relative position and velocity is developed. Finally, numerical simulations are carried out to test the proposed algorithm.

II. RELATIVE TRANSLATIONAL MOTION DYNAMICS

In this paper, relative motion of two spacecrafts in close proximity is considered. Relative orbital dynamics equations are written in the Local-vertical-Local-Horizontal (LVLH) reference frame attached to the leader and the orthogonal body frame fixed to the center of mass of the follower. X-axis points radially outward of leader's orbit, Y-direction perpendicular to X along its direction of motion and Z completes the right handed co-ordinate system. The relative orbit position vector $\boldsymbol{\rho}$ is expressed as $\boldsymbol{\rho} = [\mathrm{x\ y\ z}]^T$. The motion of the follower with respect to the leader is described in the LVLH

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frame by nonlinear Clohessy-Wiltshire equations [4]

$$\ddot{x} - x\dot{\theta}^2(1 + \frac{2r_c}{p}) - 2\dot{\theta}(\dot{y} - \frac{y\dot{r_c}}{r_c}) = w_x \quad (1a)$$

$$\ddot{y} + 2\dot{\theta}(\dot{x} - \frac{x\dot{r_c}}{r_c}) - y\dot{\theta}^2(1 - \frac{r_c}{p}) = w_y \quad (1b)$$

$$\ddot{z} + z\dot{\theta}^2\frac{r_c}{p} = w_z \quad (1c)$$

where p is semilatus rectum, r_c is orbit radius and $\dot{\theta}$ is the true anomaly rate of the of the leader. w_x, w_y, and w_z are acceleration disturbances which are modeled as zero mean Gaussian white-noise processes, with variances given by σ_x^2, σ_y^2 and σ_z^2 respectively. For circular orbit of the leader relative equations of motion reduce to the simple form known as the CW equations :

$$\ddot{x} - 2n\dot{y} - 3n^2x = w_x \quad (2a)$$

$$\ddot{y} + 2n\dot{x} = w_y \quad (2b)$$

$$\ddot{z} + n^2z = w_z \quad (2c)$$

where n = $\dot{\theta}$ is the mean motion.

III. Relative Rotational Motion Dynamics

In this section, the attitude kinematics equation of motion are briefly reviewed. A detailed derivation can be found in [2], [5] and [6] . Spacecraft attitude can be represented by various parameters like Euler angles, Rodrigues parameters, modified Rodrigues parameters and quaternions. Quaternions are ideal and are the most widely used parameterization for attitude estimation. A quaternion $\mathbf{q}$ has a three-vector part, $[q_1\ q_2\ q_3]^T$, and a scalar part q_4, with

$$\boldsymbol{\varrho} \equiv [q_1 \quad q_2 \quad q_3]^T = \hat{\mathbf{e}}sin(\vartheta/2) \quad (3a)$$

$$q_4 = cos(\vartheta/2) \quad (3b)$$

where $\hat{\mathbf{e}}$ and ϑ are the axis of rotation and angle of rotation respectively. The quaternion satisfies a unit norm constraint. The attitude matrix expressed in quaternions is given by

$$A(\mathbf{q}) = \Xi^T(\mathbf{q})\psi(\mathbf{q}) \quad (4)$$

with

$$\Xi(\mathbf{q}) \equiv \begin{bmatrix} q_4I_{(3\times3)} + [\varrho\times] \\ -\varrho^T \end{bmatrix} \quad (5a)$$

$$\psi(q) \equiv \begin{bmatrix} q_4I_{(3\times3)} - [\varrho\times] \\ -\varrho^T \end{bmatrix} \quad (5b)$$

$$[\boldsymbol{\varrho}\times] \equiv \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix} \quad (5c)$$

The quaternion kinematics is given as :

$$\dot{\mathbf{q}} = \frac{1}{2}\Xi(\mathbf{q})\boldsymbol{\omega} = \frac{1}{2}\Omega(\boldsymbol{\omega})\mathbf{q} \quad (6)$$

where

$$\Omega(\boldsymbol{\omega}) = \begin{bmatrix} -[\boldsymbol{\omega}\times] & \boldsymbol{\omega} \\ -\boldsymbol{\omega}^T & 0 \end{bmatrix} \quad (7)$$

A multiplicative error quaternion is defined as:

$$\delta\mathbf{q} = \mathbf{q} \otimes \hat{\mathbf{q}} \quad (8)$$

$$\delta\boldsymbol{\varrho} = \Xi(\hat{\mathbf{q}})\mathbf{q} \quad (9)$$

$$\delta q_4 = \hat{\mathbf{q}}^T\mathbf{q} \approx 1 \quad (10)$$

The quaternion multiplication ($\mathbf{q}' \otimes \mathbf{q}$) is defined as in [5].

IV. Measurement Model : Vision based navigation system

Photogrammetry technique involves measuring objects from images or LOS measurements [2]. The attitude and position of the leader from LOS observations can be determined by following collinearity equations [7]:

$$\chi_i = -f\frac{A_{11}(X_i - x) + A_{12}(Y_i - y) + A_{13}(Z_i - z)}{A_{31}(X_i - x) + A_{32}(Y_i - y) + A_{33}(Z_i - z)} \quad (11a)$$

$$\gamma_i = -f\frac{A_{21}(X_i - x) + A_{22}(Y_i - y) + A_{23}(Z_i - z)}{A_{31}(X_i - x) + A_{32}(Y_i - y) + A_{33}(Z_i - z)} \quad (11b)$$

where i = 1,2,...,N are the total observations, (χ_i,γ_i) are the image space observations for the ith LOS, (X$_i$, Y$_i$, Z$_i$) are the known reference space locations (leader) of the ith beacon, (x, y, z) are the unknown space location of the sensor (follower) and f is the known focal length. A$_{jk}$ are the unknown coefficients of the attitude matrix A, associated to the orientation from the reference plane (leader) to the image plane (follower). The objective is to determine attitude and relative position (x,y,z) given observations (χ_i, γ_i) and (X$_i$, Y$_i$, Z$_i$). The sensor observations can be written in the following orthogonal projection:

$$\mathbf{b_i} = A\mathbf{r_i}, \qquad i = 1, 2, ..., N \quad (12)$$

where

$$\mathbf{b_i} \equiv \frac{1}{\sqrt{f^2 + \chi^2 + \gamma^2}}\begin{bmatrix} -\chi_i \\ -\gamma_i \\ f \end{bmatrix} \quad (13a)$$

$$\mathbf{r_i} \equiv \frac{1}{\sqrt{(X_i - x)^2) + (Y_i - y)^2) + (Z_i - z)^2)}}\begin{bmatrix} X_i - x \\ Y_i - y \\ Z_i - z \end{bmatrix} \quad (13b)$$

In the presence of measurement noise, (12) can be written as:

$$\tilde{\mathbf{b}}_i = A\mathbf{r}_i + \boldsymbol{v}_i \quad (14)$$

where $\tilde{\mathbf{b}}_i$ is the ith measurement, and $\boldsymbol{v}_i$ is zero mean Gaussian white noise with covariance matrix R$_i$.

V. Relative Angular Velocity Estimation

In this section an algorithm to estimate the angular velocity is developed using LOS measurements and Kalman filtering. This algorithm was proposed by P. Singla et al. in [3] to determine the spacecraft body angular rates from the star tracker body measurements. We have extended this algorithm to directly obtain the relative angular rates from

LOS observations of leader. This eliminates the need to know angular velocity of the leader, which requires exchange of information between the two spacecrafts, to determine the relative attitude. Using (12) the velocity of the measurement model is given by

$$\frac{d\mathbf{b}_i(t)}{dt} = \frac{d\mathbf{A}(\mathbf{t})}{dt}\mathbf{r}_i \tag{15}$$

Using the fact that

$$\frac{dA(t)}{dt} = -[\boldsymbol{\omega}\times]A(t) \tag{16}$$

we can write,

$$\frac{d\mathbf{b}_i(t)}{dt} = -[\boldsymbol{\omega}\times]\mathbf{b}_i(t) \tag{17}$$

By considering the first order Taylor series expansion for i^{th} LOS vector at time t_k and using (14) and (17),

$$\begin{aligned}\mathbf{Y}_i(k) &= \frac{1}{\Delta t}[\tilde{\mathbf{b}}_i(k) - \tilde{\mathbf{b}}_i(k-1)] \\ &= [\tilde{\mathbf{b}}_i(k-1)\times]\boldsymbol{\omega}(k-1) + \mathbf{w}_i(k) + O(\Delta t)\end{aligned} \tag{18}$$

Where the effective measurement error $\mathbf{w}_i(k)$ is a function of $\boldsymbol{\omega}(k-1)$ and is given by:

$$\mathbf{w}_i(k) = \frac{1}{\Delta t}[\boldsymbol{\upsilon}_i(k) - \boldsymbol{\upsilon}_i(k-1)] + [\boldsymbol{\omega}(k-1)\times]\boldsymbol{\upsilon}_i(k-1) \tag{19}$$

Assuming a stationary noise process $\boldsymbol{\upsilon}_i$ with isotropic measurement errors (that is $\mathbf{R}_i$ is scalar times identity matrix) and sampling interval is well within Nyquist's limit, that is, $\|\boldsymbol{\omega}\|\,\Delta t \leq \pi$, the measurement noise covariance matrix can be approximated to $\frac{2}{\Delta t^2}\mathbf{R}_i$. Accuracy of this algorithm can be improved by considering second order Taylor series expansion of LOS vector in (18). This will lead to truncation errors of magnitude O(Δt^2). Then:

$$\begin{aligned}\mathbf{Y}_i(k) &= \frac{1}{2\Delta t}[4\tilde{\mathbf{b}}_i(k-1) - 3\tilde{\mathbf{b}}_i(k-2) - \tilde{\mathbf{b}}_i(k)] \\ &= [\tilde{\mathbf{b}}_i(k-1)\times]\boldsymbol{\omega}(k-2) + \mathbf{w}_i(k) + O(\Delta t^2)\end{aligned} \tag{20}$$

where

$$\begin{aligned}\mathbf{w}_i(k) = \frac{1}{2\Delta t}&[4\boldsymbol{\upsilon}_i(k-1) - 3\boldsymbol{\upsilon}_i(k-2) - \boldsymbol{\upsilon}_i(k)] \\ &+[\tilde{\mathbf{b}}_i(k-2)\times]\boldsymbol{\upsilon}_i(k)\end{aligned} \tag{21}$$

Applying same assumptions on $\boldsymbol{\upsilon}_i$ as before, the measurement covariance matrix can be approximated to $\frac{13}{2\Delta t^2}\mathbf{R}_i$.

A. Kalman filtering for relative angular velocity estimation

The derivation of the Kalman filter formulation can be found in [8]. The state vector of the Kalman filter consists of three components of relative angular velocity, that is, $\mathbf{x} = [\boldsymbol{\omega}]$. The angular acceleration of the spacecraft is modeled by a first-order random process given by

$$\boldsymbol{\tau} = \dot{\boldsymbol{\omega}} = \boldsymbol{\eta} \tag{22}$$

where $\boldsymbol{\eta}$ is a Gaussian variable with the following properties:

$$\begin{aligned}E(\boldsymbol{\eta}) &= 0 \\ E(\boldsymbol{\eta}\boldsymbol{\eta}^T) &= \sigma^2 I_{3\times3}\end{aligned} \tag{23}$$

*1) **Propagation Equations:*** The state differential equation is given by

$$\dot{\mathbf{x}} = f\mathbf{x} + g\mathbf{w} \tag{24}$$

Equations (23) and (24) constitute the assumed dynamic model for the propagation of $\mathbf{x}$ between two sets of LOS measurements, that is,

$$\begin{aligned}\mathbf{w} &= \boldsymbol{\eta} \\ f &= 0_{3\times3} \\ g &= I_{3\times3}\end{aligned} \tag{25}$$

A discrete-time propagation, as given in [5], can be used for the covariance matrix in order to reduce the computational load. The covariance prediction is then given by

$$P^-_{k+1} = \phi\prime_k P^+_k \phi\prime^T_k + gQ\prime g^T \tag{26}$$

where, $\phi\prime_k = I_{3\times3}$ and $Q\prime = \sigma^2 I_{3\times3}\Delta t$.

*2) **Update equations:***

$$\hat{\boldsymbol{\omega}}^+_k = \hat{\boldsymbol{\omega}}^-_k + K_k(\mathbf{Y}_i(k) - H_k\hat{\boldsymbol{\omega}}^-_k) \tag{27}$$

where

$$\mathbf{Y_i}(k) = \frac{1}{2\Delta t}[4\tilde{\mathbf{b}}_i(k-1) - 3\tilde{\mathbf{b}}(k-2) - \tilde{\mathbf{b}}_i(k)] \tag{28a}$$

$$H_k = [\tilde{\mathbf{b}}_i(k-2)\times] \tag{28b}$$

$$K_k = P^-_K H^T_k(H_k P^-_k H^T_k + R'_k)^- \tag{28c}$$

$$R'_k = \frac{13}{2\Delta t^2}\mathbf{R}_i \tag{28d}$$

Above update equations are given for the second order expansion of i^{th} LOS vector. Equation (28a) and (28b) can be replaced by (18), if first order approach is to be used in the filter.

VI. Kalman Filter for Relative State estimation

In this section Kalman filtering for estimating relative attitude, position and velocity is presented. The error in relative attitude is represented by multiplicative error quaternion as shown in (8). This error vector can be written as:

$$\delta\mathbf{q} \approx \begin{bmatrix}\delta\boldsymbol{\alpha}/2 & 1\end{bmatrix} \tag{29}$$

Where $\delta\boldsymbol{\alpha}$ is small angle-error correction. The state vector of the Kalman filter consists of the error in relative attitude, relative position $\Delta\boldsymbol{\rho}$ and relative velocity $\Delta\dot{\boldsymbol{\rho}}$:

$$\mathbf{X} = \begin{bmatrix}\delta\boldsymbol{\alpha} \\ \Delta\boldsymbol{\rho} \\ \Delta\dot{\boldsymbol{\rho}}\end{bmatrix} \tag{30}$$

The rate of change of error in the quaternion is given by :

$$\delta\dot{\boldsymbol{\alpha}} = -\hat{\boldsymbol{\omega}} \times \delta\boldsymbol{\alpha} + \delta\boldsymbol{\omega} \tag{31}$$

The state differential equation is given by :

$$\dot{\mathbf{X}} = F\mathbf{X} + G\mathbf{W} \tag{32}$$

The F and G matrices are given by :

$$F = \begin{bmatrix} -[\hat{\boldsymbol{\omega}}\times] & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & I_{3\times3} \\ 0_{3\times3} & F_{32} & F_{33} \end{bmatrix}$$
$$F_{32} \equiv \begin{bmatrix} 3n^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -n^2 \end{bmatrix}$$
$$F_{33} \equiv \begin{bmatrix} 0 & 2n & 0 \\ -2n & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (33)$$
$$G = \begin{bmatrix} I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & I_{3\times3} \end{bmatrix}$$

The vector $\mathbf{W} = [\hat{\boldsymbol{\omega}}^T \quad w_x \quad w_y \quad w_z]^T$ with E $(\mathbf{W}\mathbf{W}^T)$ = $\mathbf{S}$. As $\mathbf{W}$ contains $\hat{\boldsymbol{\omega}}$, whose value varies with time, hence $\mathbf{S}$ is a function of time:

$$\mathbf{S_k} = \begin{bmatrix} E(\hat{\boldsymbol{\omega}}_k\hat{\boldsymbol{\omega}}_k^T) & 0 & 0 & 0 \\ 0_{1\times3} & \sigma_x^2 & 0 & 0 \\ 0_{1\times3} & 0 & \sigma_y^2 & 0 \\ 0_{1\times3} & 0 & 0 & \sigma_z^2 \end{bmatrix} \quad (34)$$

A. *Propagation equations*

The state vector and error covariance matrix propagation are given by :

$$\hat{\mathbf{X}}_{k+1}^- = F\hat{\mathbf{X}}_k^+ \quad (35)$$

The relative position and velocity are then:

$$\hat{\boldsymbol{\rho}}_{k+1}^- = \hat{\boldsymbol{\rho}}_k^+ + \Delta\hat{\boldsymbol{\rho}}_{k+1}^+ \quad (36a)$$
$$\dot{\hat{\boldsymbol{\rho}}}_{k+1}^- = \dot{\hat{\boldsymbol{\rho}}}_k^+ + \Delta\dot{\hat{\boldsymbol{\rho}}}_{k+1}^+ \quad (36b)$$
$$(36c)$$

The quaternion propagation is given by:

$$\hat{\mathbf{q}}_{k+1}^- = \Theta(\hat{\boldsymbol{\omega}}_k^+)\hat{\mathbf{q}}_k^+ \quad (37)$$

with

$$\Theta(\hat{\boldsymbol{\omega}}_k^+) = \begin{bmatrix} \zeta_k^+ I_{3\times3} - [\hat{\Psi}_k^+\times] & \hat{\Psi}_k^+ \\ -\hat{\Psi}_k^{+T} & \zeta_k^+ \end{bmatrix}$$
$$\zeta_k^+ = cos\Big(\frac{1}{2}\left\|(\hat{\boldsymbol{\omega}}_k^+)\right\|\Delta t\Big) \quad (38)$$
$$\hat{\Psi}_k^+ = \frac{sin\big(\frac{1}{2}\left\|(\hat{\boldsymbol{\omega}}_k^+)\right\|\Delta t\big)\omega_k^+}{\left\|\hat{\boldsymbol{\omega}}_k^+\right\|}$$

The error covariance matrix is propagated as:

$$P_{k+1}^- = \phi_k P_k^+ \phi_k + GQ_kG^T \quad (39)$$

Where ϕ_k and Q_k are obtained by Van Loan method [9], a numerical method for fixed parameter systems, which include a constant sampling interval time, Δt, time invariant state, F and G, and covariance matrix S_k.

B. *Update equations*

The Kalman gain K_k at time t_k is given by

$$K_k = P_k^- H_k^T(\hat{X}_k^-)[H_k(\hat{X}_k^-)P_k^- H_k^T(\hat{X}_k^-) + S_k]^- \quad (40)$$

where the sensitivity matrix is given by:

$$H_k(\hat{X}_k^-) = \begin{bmatrix} [A(\hat{\mathbf{q}}_k^-)\hat{\mathbf{r}}_1\times] & \frac{\partial\hat{\mathbf{b}}_1^-}{\partial\hat{\rho}_k^-} & 0_{3\times3} \\ \vdots & \vdots & \vdots \\ [A(\hat{\mathbf{q}}_k^-)\hat{\mathbf{r}}_N\times] & \frac{\partial\hat{\mathbf{b}}_N^-}{\partial\hat{\rho}_k^-} & 0_{3\times3} \end{bmatrix} \quad (41)$$

Where $\hat{\mathbf{r}}_i^-$ is given by by (13) and evaluated at $\hat{\boldsymbol{\rho}}^-$ and the partial matrix

$$\frac{\partial\hat{\mathbf{b}}_i^-}{\partial\hat{\rho}^-} = \frac{A(\hat{\mathbf{q}}^-)}{[(X_i-x)^2+(Y_i-y)^2+(Z_i-z)^2]^{3/2}}\frac{\partial\hat{\mathbf{r}}_i^-}{\partial\hat{\rho}^-}$$

with $\frac{\partial\hat{\mathbf{r}}_i^-}{\partial\hat{\rho}^-}$ given by (42).

The state and covariance update are then given by :

$$P_k^+ = [I_{3\times3} - K_kH_k(\hat{\mathbf{X}}_k^-)]P_k^- \quad (43a)$$
$$\hat{\mathbf{X}}_k^+ = K_k[\tilde{y}_k - h_k(\hat{\mathbf{q}}_k^-)] \quad (43b)$$

where

$$h_k(\hat{\mathbf{q}}_k^-) = \begin{bmatrix} A(\hat{\mathbf{q}}_k^-)\hat{\mathbf{r}}_1 \\ A(\hat{\mathbf{q}}_k^-)\hat{\mathbf{r}}_2 \\ \vdots \\ [A(\hat{\mathbf{q}}_k^-)\hat{\mathbf{r}}_N \end{bmatrix} \quad (44a)$$
$$\tilde{y}_k = \begin{bmatrix} \tilde{b}_1(k) \\ \tilde{b}_2(k) \\ \vdots \\ \tilde{b}_N(k) \end{bmatrix} \quad (44b)$$

The relative position and velocity updates are:

$$\hat{\boldsymbol{\rho}}_k^+ = \hat{\boldsymbol{\rho}}_k^- + \Delta\hat{\boldsymbol{\rho}}_k^+ \quad (45a)$$
$$\dot{\hat{\boldsymbol{\rho}}}_k^+ = \dot{\hat{\boldsymbol{\rho}}}_k^- + \Delta\dot{\hat{\boldsymbol{\rho}}}_k^+ \quad (45b)$$

VII. Simulations

Fig. 1 shows a flowchart of the proposed algorithm for estimating relative attitude and position. Numerical simulations were performed to test the performance of the EKF for estimating relative attitude, angular velocity, position and translational velocity. For the leader, a circular orbit with orbital radius of 7,078,000 m is considered. A bounded relative orbit is used. The simulation time for the relative motion of both spacecrafts is 90 minutes and the sampling interval is 0.4 s. The initial quaternion (in rad) and angular velocity (in rad/s) for the relative attitude motion is given by:

$$\mathbf{q}_0 = [\sqrt{2}/2 \quad 0 \quad 0 \quad \sqrt{2}/2]^T \quad (46a)$$
$$\boldsymbol{\omega}_0 = [-.002 \quad .0011 \quad .0022]^T \quad (46b)$$

The initial condition for the vector $\mathbf{X}$ in appropriate SI units (rad, m and m/s) is:

$$\mathbf{X} = [\mathbf{q}_0^T \quad 400 \quad 0 \quad 0 \quad 0 \quad -.6361 \quad 0]^T \quad (47)$$

$$\frac{\partial \hat{\mathbf{r}}_i^-}{\partial \hat{\rho}^-} = \begin{bmatrix} -[(Y_i-\hat{y})^2+(Z_i-\hat{z})^2] & (X_i-\hat{x})(Y_i-\hat{y}) & (X_i-\hat{x})(Z_i-\hat{z}) \\ (X_i-\hat{x})(Y_i-\hat{y}) & -[(X_i-\hat{x})^2+(Z_i-\hat{z})^2] & (Y_i-\hat{y})(Z_i-\hat{z}) \\ (X_i-\hat{x})(Z_i-\hat{z}) & (Y_i-\hat{y})(Z_i-\hat{z}) & -[(X_i-\hat{x})^2+(Y_i-\hat{y})^2] \end{bmatrix} \tag{42}$$

Three beacons are assumed to exist on the leader spacecraft:

$$X_1 = 1m \quad Y_1 = .01m \quad Z_1 = .01m \tag{48a}$$

$$X_2 = .01m \quad Y_2 = .5m \quad Z_2 = .86m \tag{48b}$$

$$X_3 = .01m \quad Y_3 = -.5m \quad Z_3 = .86m \tag{48c}$$

Measurement updates in the filter are only used when the beacons are within the field of view of the sensor. Measurement model is simulated with a standard deviation of .0003 deg. Initialization of EKF is carried out by using a nonlinear least squares routine from the synthetic measurements. This gives initial relative angular velocity with an error of 2 deg/hr for each axis. The initial errors in relative position and velocity for each axis are 1 m and .01 m/s respectively. The initial covariance sub matrix for relative attitude, angular velocity, position and velocity is taken to be a diagonal matrix with equal elements. Two sets of simulations are carried out for the same initial conditions of relative motion. Test Case 1 is based on the second order approach for estimating relative angular velocity while Test Case 2 uses first order approach to estimate relative angular velocity. Fig. 2, Fig. 4, Fig. 6 and Fig. 7 are the plots obtained for Test Case 1.

Fig. 2 shows the relative position errors with respective 3σ bounds obtained from the EKF covariance matrix. The relative position is known within .02 m for each axis and error converges to zero in 15 s. The relative velocity errors remain below .01 m/s as shown in Fig. 3 and error achieves convergence in approximately 6 s. The relative angular velocity errors are within .001 rad/s as shown in Fig. 4. The relative attitude error remains well within .2 deg and estimate errors for attitude are larger than actual error as can be seen from 3σ bounds in Fig.6. All errors remain within 3σ bounds indicating that EKF is working properly. Fig. 3 and Fig. 5 correspond to Test Case 2. Relative errors in position and velocity are increased while still remaining within 3σ bounds. Convergence time for errors also increase to 54 s for relative position and to 35 s for relative velocity.

It can therefore be concluded that using second order approach for estimating relative angular velocity gives more accurate results as compared to the first order approach. The possible reason for this is the use of two past measurements along with the measurement at present time in the second order approach and also a decrease in the truncation error. Apart from this, the accuracy of the Kalman filter depends upon the accuracy of the dynamic model and tuning of process noise matrix. This means that between two sets of measurements, accumulation of model error can take place in the estimates. Estimates also depends on the number of beacons on the leader and their spread, so as to ensure minimum number of beacons in the field of view of VISNAV sensor. Also, CW equations can only be applied when the distance between the leader and follower is under 1 Km.

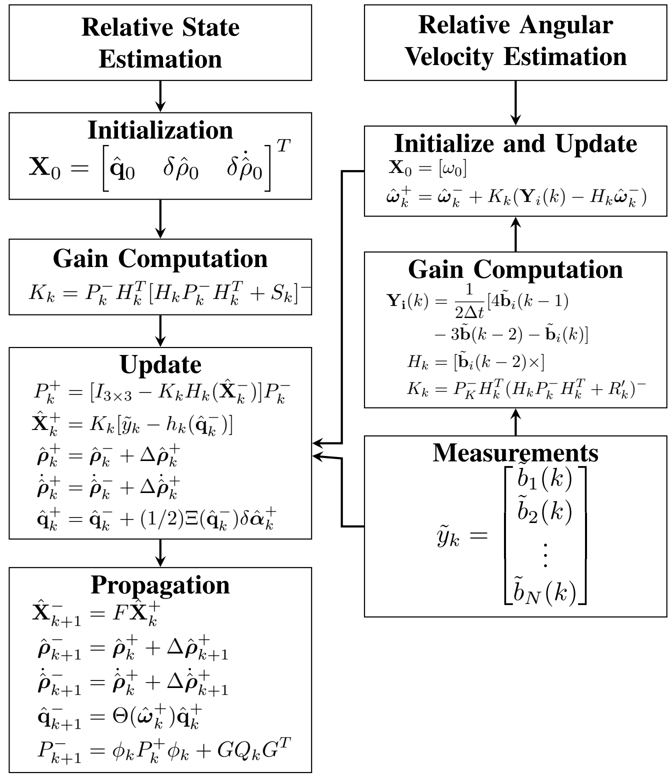

Fig. 1: Flow Chart of the Proposed Algorithm

VIII. CONCLUSION

An algorithm for relative spacecraft navigation is developed to estimate relative state using only LOS vectors. Extended Kalman filter is designed for estimating relative angular velocity, attitude, position and velocity. Relative angular velocity can be estimated independent of attitude and gyro measurements from the spacecrafts. A combined EKF is then designed to estimate relative position, velocity and attitude. In the future work, gravitational perturbations, solar pressure and higher order nonlinear effects can be included in the dynamic model of Kalman filter. A combination of generalized Clohessy and Wiltshire equations and Gauss variational equations can be used for this purpose. In this paper relative motion of only two spacecrafts is considered. Numerical simulations involving extended leader follower network can be carried on the similar lines.

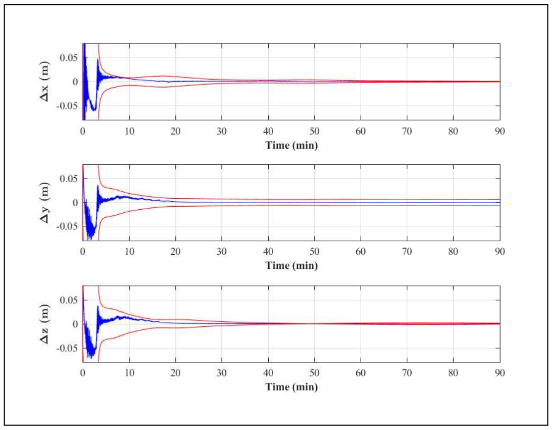

Fig. 2: Relative position errors and 3σ bounds for Test Case 1

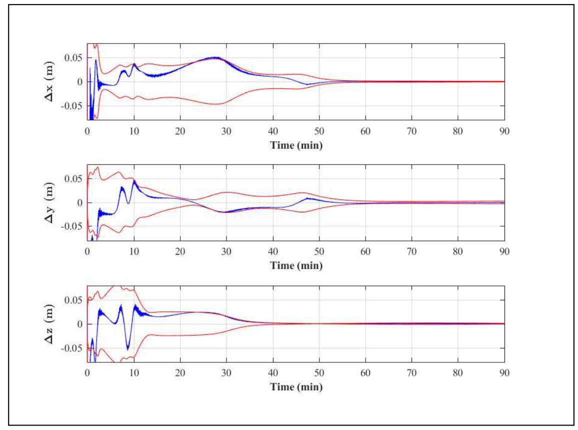

Fig. 3: Relative position errors and 3σ bounds for Test Case 2

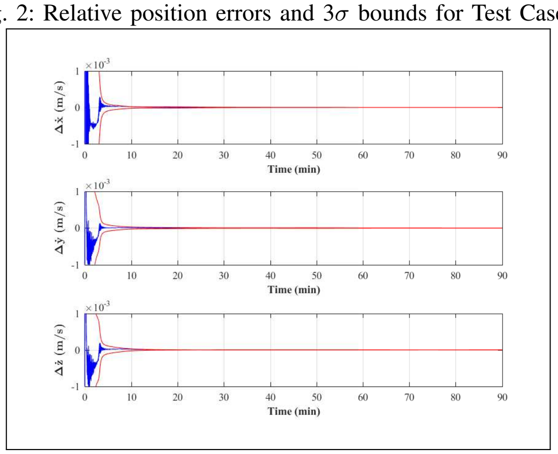

Fig. 4: Relative velocity errors and 3σ bounds for Test Case 1

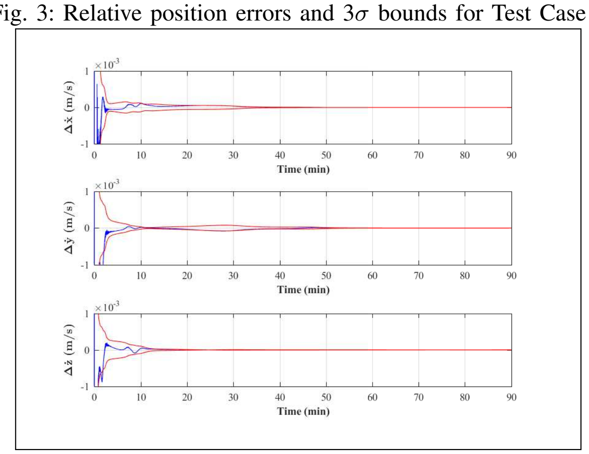

Fig. 5: Relative velocity errors and 3σ bounds for Test Case 2

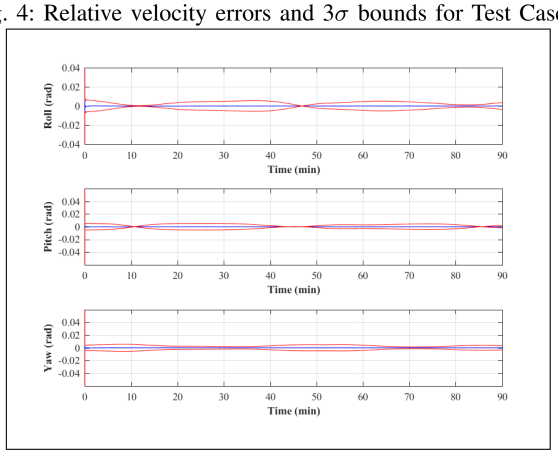

Fig. 6: Relative attitude errors and 3σ bounds for Test Case 1

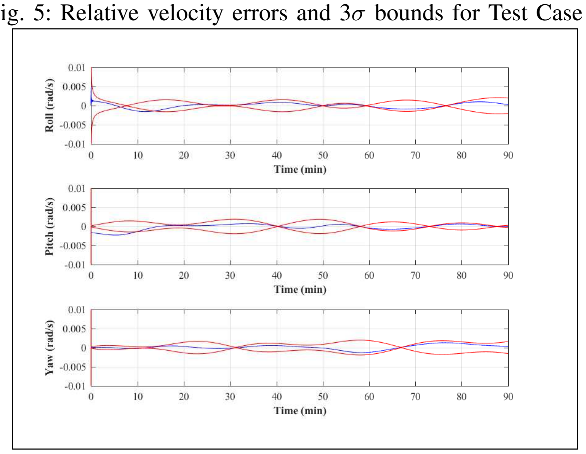

Fig. 7: Relative angular velocity errors and 3σ bounds for Test Case 1

REFERENCES

[1] Heaton, Andrew F, Howard, Richard T., Pinson, Robin M., Orbital Express AVGS Validation and Calibration for Automated Rendezvous, AIAA Astrodynamics Specialist Conference, 17-2 Aug. 2008, Honolulu, HI, United States

[2] Son-Goo Kim, John L. Crassidis, Yang Cheng, Adam M. Fosbury, John L. Junkins, Kalman Filtering for Relative Spacecraft Attitude and Position Estimation,Journal of Guidance, Control, and Dynamics, vol. 30, Issue 1, 2007/1, pp. 133-143

[3] Puneet Singla, John L Crassidis, John L Junkins, Spacecraft angular rate estimation algorithms for star tracker-based attitude determination, Advances in the Astronautical Sciences, 2003, vol. 114, pp. 1303-1316

[4] Howard D. Curtis, Orbital Mechanics for Engineering Students, 3rd ed, Butterworth-Heinemann, ch. 7

[5] F Landis Markley, John L Crassidis, Fundamentals of spacecraft attitude determination and control, Springer, 2014/5/31

[6] Puneet Singla, D Todd Griffith, JL Crassidis, JL Junkins, Attitude determination and autonomous on-orbit calibration of star tracker for the gifts mission, 2002/1/27, American Astronomical Society, AAS 02-101

[7] Puneet Singla, Kamesh Subbarao, John L Junkins, Structured Model Reference Control for Vision Base Spacecraft Rendezvous And Docking, AAS 03-103

[8] Robert Grover Brown, Patrick Y. C. Hwang, Introduction to Random Signals and Applied Kalman Filtering with Matlab Exercises, 4th ed, Feb 2012, ch. 4 and ch. 7

[9] Van Loan, C.F., Computing Integrals Involving the Matrix Exponential, IEEE Transactions on Automatic Control, vol. AC-23, No.3, June 1978, pp. 396-404