Consensus of Networked Euler-Lagrange System Under Measurement Bias

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by

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Abstract

This project addresses the problem of distributed consensus tracking of multi agent systems. A Networked Euler-Lagrange (EL) system is considered in this project. A class of mechanical systems including autonomous vehicles, spacecraft in formation, robotic manipulators, and walking robots are Lagrangian systems. We focus on fully-actuated Lagrangian systems. We propose a distributed consensus tracking algorithm for certain EL system for the scenario when each follower inaccurately measures its neighbors' positions and the leader's position if it has access to the leader. This controller ensures that bias estimation errors are converged exponentially to zero without any knowledge of upper bound on the bias while achieving asymptotic convergence of both position and velocity to that of leader's trajectory. An adaptive control law is derived based on Lyapunov analysis and initial excitation condition on regressor to estimate the bias. The proposed algorithms ensure that the velocities and positions converge to leader's trajectory exponentially while also ensuring bias estimation errors goes to zero. Simulation results corroborate our theoretical findings.

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Chapter 1

Introduction and Literature Survey

In recent years there have been a growing interest in the field of swarm robotics. Advances in technology have enabled cooperative multi-agent systems to have many practical applications. Spacecraft rendezvous, rigid body attitude synchronization and formation flight are examples of scenarios where the agents need to update their (information) states to reach a consensus. Spacecraft formation flying is one of the most important technological challenges for modern day space agencies with application to areas like synthetic aperture radars and deep space exploration [15]. These missions require that spacecraft maintain a desired relative position and attitude at all times. In synchronization problems *consensus* is the significant objective and implies that all the agents reach an agreement on a common value by locally interacting with their neighbors. In distributed multi-agent coordination problems (distributed algorithm allows the agents to execute control law without requiring information of the network as a whole), point models are generally considered due to their simplicity but are not realistic. Euler-Lagrange equations can be used to model a large class of aero-mechanical systems including autonomous vehicles and spacecraft in formation [12]. Networked Lagrangian systems are studied in detail in [12], where the authors propose consensus algorithms accounting for actuator saturation and for unavailability of measurements of generalized coordinates. In [9] formation dynamics of spacecraft formation is discussed, describing the dynamics in Euler-Lagrangian form.

Distributed and model independent algorithms for directed networks in the presence of bounded disturbance is addressed in [4]. In [2] a control law to achieve finite time coordinated control for 6DOF spacecraft formation is developed. However, this algorithm is model dependent and requires the knowledge of self states of the agents, and further the gravitational and centrifugal forces acting on them. A model dependent control law is designed in [10] using contraction analysis for synchronization of spacecraft. In [11], a synchronization controller for attitude and position control of a spacecraft formation is designed which rely on all to all communication topology. An algorithm for tracking of Lagrangian systems using only position measurements is developed in [8] by encompassing a distributed observer to estimate unknown velocity of the agents. An output feedback structured model reference adaptive control (MRAC) law has been developed for spacecraft rendezvous in [7]. However, their control law works well only in the presence of bounded disturbances and measurement errors. In [3], the coordination control problem of heterogeneous first and second order multi-agent systems with external disturbances is considered, but the disturbances are assumed to be $\mathbb{L}^2$ bounded. In [14], a composite consensus control strategy is proposed for second-order multi-agent systems with mismatched bounded disturbances.

In [17] a novel adaptive control law is introduced. This law makes use of both prediction and tracking error to estimate the unknown parameters of the system. The idea behind the composite adaption is based on the observation that the parameter uncertainty gets reflected in both the tracking error and prediction error and so it is desirable to extract the parameter information from both the sources. For uncertain EL systems, conventional adaptive control laws requires the regressor function to be persistently exciting (PE) for parameter convergence. In [16] a PI-like (Proportional-Integral controllers) parameter update law is designed, which guarantees parameter convergence without requiring the PE condition. However, a milder initial excitation (IE) condition is imposed on the regressor to ensure parameter convergence.

A second-order super-twisting sliding mode controller is developed in [19] for consensus tracking of the leader-follower multi agent systems in the presence of uncertainties and external disturbances. This approach can ensure the finite-time consensus only if the bounds on the disturbances and parameter uncertainties are known to the agents. In [18] a continuous sliding mode tracking protocol with an adaptive mechanism is developed for the consensus tracking problem of second-order nonlinear multiagent systems with bounded disturbance and bounded actuator fault. [21] proposes adaptive algorithm which make use of both instantaneous state data and past measurements for the adaptation process. This scheme ensures parameter estimation errors to converge to zero exponentially subject to the satisfaction of a finite excitation condition, which is a relaxation when compared to the persistent excitation condition.

In the aforementioned literature on consensus with errors, adaptive control algorithms in the presence of an upper bound on disturbances and stochastic errors have been studied. But what happens to consensus in the presence of measurement errors with unknown bounds? The current work addresses this problem. Further, strategies for handling disturbance do not usually fare well for the case of measurement errors simply due to the fact that the measurement errors scale with the control gain while disturbances external to the system do not. This makes ensuring bounded trajectories with constant measurement bias a much harder problem than the disturbance robustness case.

There are only a handful of relevant contributions in the domain of measurement bias errors that are known to the authors. In [20] a leader-following consensus control for double integrator multi agent system is proposed where each follower inaccurately measures its neighbors' positions which are corrupted by noises. This control law ensures that the tracking consensus is achieved in mean square, for both fixed and switching directed topologies. [1] proposes an adaptive control law in the presence of unknown constant bias for a double integrator system. This controller only achieve convergence within a neighborhood of consensus, the size of which is dictated by the bias magnitudes. [5] addresses the problem of accommodating unknown sensor bias in a direct MRAC setting for state tracking using state feedback. Motivated by the above work, we have developed a novel composite adaptive controller based on Lyapunov analysis for the scenario when each follower inaccurately measures its neighbors' positions and the leader's position if it has access to the leader. The measured positions are corrupted by the presence of a constant bias. The main contribution of this work is that bias estimation errors are ensured to exponentially converge to zero without any knowledge of upper bound on the bias while achieving asymptotic convergence of both position and velocity to that of leader's trajectory exponentially.

This thesis is organized as follows. Second chapter provides a brief introduction on basic concepts of graph theory. In third chapter a distributed, model independent control law for synchronization of networked Euler Lagrange system with biased measurements is designed. A strict assumption of all-to-all leader connectivity is made here. This algorithm ensures that the velocities converge to that of leader exponentially while the positions converge only to a bounded neighborhood of the leader positions. Next, we have attempted to reduce this error appreciably to ensure exponential convergence of position, velocity and estimated bias using a new type of composite adaptive controller. Simulation are performed for a six spacecraft formation in both the chapters. In chapter five, a distributed consensus tracking algorithm for certain EL system is proposed for the scenario when each follower inaccurately measures its neighbors' positions and the leader's position if it has access to the leader and hence the all-to-all connectivity condition is relaxed here. This controller ensures that bias estimation errors are converged exponentially to zero without any knowledge of upper bound on the bias while achieving asymptotic convergence of both position and velocity to that of leader's trajectory.

Chapter 2

Preliminaries

In this chapter we present several notations, lemmas, assumptions and an introduction on graph theory for subsequent use.

2.1 Mathematical Notations

Given a vector $\mathbf{x} = [x_1, ..., x_n]^T \in \mathbb{R}^n$, $\text{sgn}(\mathbf{x}) = [\text{sgn}(x_1), ..., \text{sgn}(x_n)]^T$, where $\text{sgn}(\cdot)$ is the standard signum function, $\mathbf{1}_n = [1, .., 1]^T$ and $\mathbf{0}_n = [0, ..., 0]^T$. One-norm and Euclidean norm of a vector $\mathbf{x}$ are denoted by $||\mathbf{x}||_1 = \sum_{i=1}^{n} |x|_i$ and $||\mathbf{x}|| = (\sum_{i=1}^{n} |x|_i^2)^{\frac{1}{2}}$ respectively. A Diagonal matrix with diagonal elements as $d_1, d_2, ..., d_n$ is represented by $\text{diag}(d_1, ..., d_n)$ and a block diagonal matrix with diagonal matrices $B_1, ..., B_n$ is represented by $\text{blkdiag}(B_1, ..., B_n)$. A $n \times n$ identity matrix and zero matrix is denoted by I_n and $\mathbf{0}_n$ respectively. We use $\otimes$ to denote Kronecker product. For a matrix A, its i^{th}, maximum and minimum eigenvalues are respectively denoted by $\lambda_i(A)$, $\lambda_{max}(A)$ and $\lambda_{min}(A)$.

2.2 Graph Theory

Consider a multi-agent system with n agents interacting with each other through a communication or sensing network or a combination of both. This network is modeled as either *undirected* or *directed* graph. We define the graph, $\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} \triangleq 1, ..., n$ is a node set and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is an edge set of nodes, called edges [13]. An edge (i, j) in the edge set of a directed graph signifies that agent j can obtain information from agent i but not vice-versa. If an edge $(i, j) \in \mathcal{E}$, then node i is a neighbor of node j. The set of neighbors of node i is denoted by $\mathcal{N}_i$. In an undirected graph the pair of nodes are unordered, where the edge (i, j) denotes that agents i and j can obtain information from each other, i.e. $(j, i) \in \mathcal{E} \Leftrightarrow (i, j) \in \mathcal{E}$. A weighted graph associates a weight with every edge in the graph. An undirected graph is connected if there is an undirected path between every pair of distinct nodes [13]. The *adjacency* matrix, $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$, is defined such that a_{ij} is a positive weight if $(j,i) \in \mathcal{E}$ and $a_{ij} = 0$ if $(j,i) \neq \mathcal{E}$. Since no self edges are present, $a_{ii} = 0$. For an undirected graph, $\mathcal{A}$ is symmetric. The *degree* matrix of the graph $\mathcal{G}$ is, $\mathcal{D} = \text{diag}(\sum_{j=1}^{n} a_{1j}, ..., \sum_{j=1}^{n} a_{nj}) \in \mathbb{R}^{n\times n}$. *Laplacian* matrix, $\mathcal{L} \triangleq [l_{ij}] \in \mathbb{R}^{n\times n}$, is then defined as

$$\mathcal{L} = \mathcal{D} - \mathcal{A}$$

$$l_{ii} = \sum_{j=1, j\neq i}^{n} a_{ij}, \qquad l_{ij} = -a_{ij}, i \neq j \tag{2.1}$$

$\mathcal{L}$ is symmetric for undirected graphs and since $\mathcal{L}$ has zero row sums, 0 is an eigenvalue of $\mathcal{L}$ with an associated eigenvector $\mathbf{1}_n$. Laplacian matrix is diagonally dominant and has non negative diagonal entries [13]. Note that, $\mathcal{L}\mathbf{x}$ is a column stack vector of $\sum_{j=1}^{n} a_{ij}(x_i - x_j)$, where $\mathbf{x} = [x_1, ..., x_n]^T \in \mathbb{R}^n$.

For a leader follower network, we let the leader be denoted by 0 and followers by nodes $1, ..., n$. The Laplacian matrix of the followers is denoted by $\mathcal{L}$. The communication between the leader and a follower is unidirectional with the leader issuing the communication. The edge weight between the leader follower is denoted by $a_{i0}, i \in \mathcal{V}$. If the i^{th} follower is connected to the leader then $a_{i0} > 0$ and 0 otherwise. We define $\bar{A} = \text{diag}(a_{10}, ...a_{n0})$.

2.3 Lemmas

Below are the lemmas for subsequent use for a leader-follower networked system.

Lemma 1. *[13] If atleast one follower is connected to the leader, then $\mathcal{L} + \bar{A}$ is positive definite.*

Lemma 2. *[13] If a symmetric matrix $H > 0 \; \forall \, \boldsymbol{x} \in \mathbb{R}^n$, then*

$$\lambda_{min}(H)\, \|x\|^2 \leq \boldsymbol{x}^T H \boldsymbol{x} \leq \lambda_{max}(H)\, \|x\|^2 \tag{2.2}$$

Lemma 3. *[13] If graph $\mathcal{G}$ is undirected and connected, then $\mathcal{L}$ has following properties:*

1. *For any $\boldsymbol{x} \in \mathbb{R}^n$, $\boldsymbol{x}^T \mathcal{L} \boldsymbol{x} = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}(x_i - x_j)^2$ which implies that $\mathcal{L}$ is positive semidefinite*

2. *$\mathcal{L}\boldsymbol{x} = 0$ or $\boldsymbol{x}^T \mathcal{L} \boldsymbol{x} = 0$ if and only if $x_i = x_j$ for all $i, j = 1, ..., n$*

3. *Let $\lambda_i(\mathcal{L})$ be the i^{th} eigenvalue of $\mathcal{L}$ with $\lambda_1(\mathcal{L}) \leq \lambda_2(\mathcal{L}) \leq \cdots \leq \lambda_n(\mathcal{L})$, so that $\lambda_1(\mathcal{L}) = 0$. Then, $\lambda_2(\mathcal{L})$ is the algeberic connectivity, which is positive if and only if the undirected graph is connected. The algebraic connectivity quantifies the convergence rate of consensus algorithms*

Lemma 4 (Barbalat's Lemma)**.** *[6] If, for a vector-valued function, $f(\cdot) : \mathbb{R} \rightarrow \mathbb{R}^n$ the following conditions hold true,*

1. *$lim_{t\rightarrow\infty} \int_0^t f(\tau)d\tau$ exists and is finite*

2. *$f(t)$ is uniformly continuous*

then, $lim_{t\rightarrow\infty} f(t) = 0$.

Corollary 4.1. *If, for a vector-valued function, $f(\cdot) : [0, \infty) \rightarrow \mathbb{R}^n$ the following two conditions hold true,*

1. *$f(x) \in \mathbb{L}^\infty \cap \mathbb{L}^p$ for any $p \in [1, \infty)$ and,*

2. *$f'(x) \in \mathbb{L}^\infty$*

then, $lim_{x\rightarrow\infty} f(x) = 0$

Lemma 5. *[12] Let $\boldsymbol{x}$ and $\boldsymbol{y}$ be any two vectors in $\mathbb{R}^n$, $\boldsymbol{A} \in \mathbb{R}^{n\times n}$ be a matrix. Then,*

$$\boldsymbol{x}^T sgn(\boldsymbol{x}) = ||\boldsymbol{x}||_1 \tag{2.3}$$

$$||\boldsymbol{x}||_1 \geq ||\boldsymbol{x}|| \tag{2.4}$$

$$|\boldsymbol{x}^T \boldsymbol{A}\boldsymbol{y}| \leq ||\boldsymbol{x}|| \, ||\boldsymbol{A}|| \, ||\boldsymbol{y}|| \tag{2.5}$$

Chapter 3

Attempt 1 for Consensus Tracking

This chapter addresses the problem of distributed coordination control of spacecraft formation. It is assumed that the agents measure relative positions of each other with a non-zero, unknown constant sensor bias. The objective of coordinated tracking is that a group of followers intercepts a dynamic leader with local interaction [12]. All the followers converge to within a neighborhood of consensus, the size of which is dictated by the bias magnitudes. Next chapter will target on reducing the bound on consensus error.

3.1 Spacecraft Relative Orbital Dynamics

For a leader follower spacecraft formation, relative translational orbital dynamics equations are described in [9]. The leader orbit frame has its origin located in the centre of mass of the leader spacecraft. The $\mathbf{e}_r$ axis is parallel to $\mathbf{r}_l$ (vector joining the center of the earth and the leader) and $\mathbf{e}_h$ axis is parallel to the orbit momentum vector which points in the orbit normal direction. The $\mathbf{e}_\theta$ axis completes the right handed orthogonal frame. Non-linear relative motion dynamics for spacecraft in formation is given by (3.1) :

$$\ddot{x} - 2\dot{\theta}\dot{y} + \left(\frac{\mu}{r_f^3} - \dot{\theta}^2\right)x - \ddot{\theta}y + \mu\left(\frac{r_l}{r_f^3} - \frac{1}{r_l^2}\right) = \frac{\tau_x}{m_f} \tag{3.1a}$$

$$\ddot{y} + 2\dot{\theta}\dot{x} + \ddot{\theta}x + \left(\frac{\mu}{r_f^3} - \dot{\theta}^2\right)y = \frac{\tau_y}{m_f} \tag{3.1b}$$

$$\ddot{z} + \frac{\mu}{r_f^3}z = \frac{\tau_z}{m_f} \tag{3.1c}$$

where $\mathbf{r}_f$ is the orbit radius of the follower and $\dot{\theta}$ is the true anomaly rate of the of the leader. $\boldsymbol{\tau}$ is the actuator force of the follower. $\mathbf{p} = \begin{bmatrix} x & y & z \end{bmatrix}^T$ is the relative position between the leader and follower in leader orbit reference frame. m_f and m_l are the masses of the follower and leader respectively and $\mu = GM_e$, where M_e is the mass of the earth.

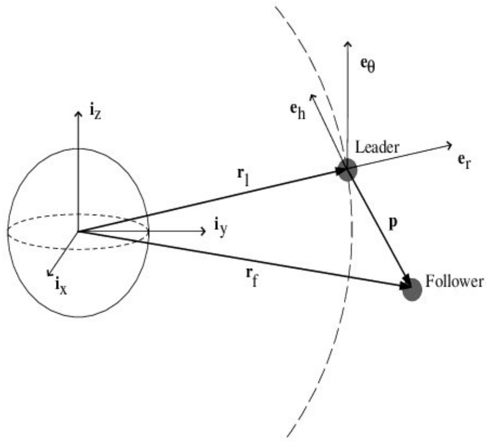

Figure 3.1: Earth centered initial frame $(\mathbf{i_X}, \mathbf{i_y}, \mathbf{i_Z})$ and Leader orbit reference frame $(\mathbf{e_r}, \mathbf{e}_\theta, \mathbf{e_h})$[9]

(3.1) can be written in the Euler Lagrangian form for the i^{th} follower as,

$$\mathbf{M}_i\ddot{\mathbf{q}}_i + \mathbf{C}_i(\dot{\theta})\dot{\mathbf{q}}_i + \mathbf{g}_i(\dot{\theta}, \ddot{\theta}, \mathbf{q}_i) = \boldsymbol{\tau}_i \tag{3.2}$$

where

$$\mathbf{M}_i = \begin{bmatrix} m_i & 0 & 0 \\ 0 & m_i & 0 \\ 0 & 0 & m_i \end{bmatrix} \tag{3.3}$$

$$\mathbf{C}_i = \begin{bmatrix} 0 & -2m_i\dot{\theta} & 0 \\ -2m_i\dot{\theta} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3.4}$$

$$\mathbf{g}_i = m_i \begin{bmatrix} \left(\frac{\mu}{r_f^3} - \dot{\theta}^2\right)x_i - \ddot{\theta}y_i + \mu\left(\frac{r_l}{r_f^3} - \frac{1}{r_l^2}\right) \\ \ddot{\theta}x_i + \left(\frac{\mu}{r_f^3} - \dot{\theta}^2\right)y_i \\ \frac{\mu z_i}{r_f^3} \end{bmatrix} \tag{3.5}$$

Here, $\mathbf{q}_i = \begin{bmatrix} x_i & y_i & z_i \end{bmatrix}^T$ and $\dot{\mathbf{q}}$ is the relative position and relative translational velocity of the i^{th} agent with respect to the leader in leader orbit reference frame. Define $\mathbf{q} \triangleq [q_1, ..., q_n]^T$, $\dot{\mathbf{q}} \triangleq [\dot{\mathbf{q}}_1, ..., \dot{\mathbf{q}}_n]^T$, $\boldsymbol{\tau} \triangleq [\boldsymbol{\tau}_1, ..., \boldsymbol{\tau}_n]^T$, $\mathbf{M} \triangleq \text{diag}(\mathbf{M}_1, ..., \mathbf{M}_n)$, $\mathbf{C} \triangleq \text{diag}(\mathbf{C}_1, .., \mathbf{C}_n)$ and $\mathbf{g} \triangleq [\mathbf{g}_1, ..., \mathbf{g}_n]^T$

3.2 Problem Formulation

We are interested in formation flight of spacecraft described by the following Euler-Lagrange equation,

$$\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + \mathbf{g}_i(\mathbf{q}_i) = \boldsymbol{\tau}_i, \qquad i = 1, ..., n \tag{3.6}$$

where $\mathbf{q}_i \in \mathbb{R}^p$ is the relative position vector of the i^{th} agent with respect to the leader, $\mathbf{M}_i(\mathbf{q}_i) \in \mathbb{R}^{p\times p}$ is the symmetric positive definite inertia matrix, $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i$ is the vector of Coriolis and centrifugal torques, $\mathbf{g}_i(\mathbf{q}_i)$ is the vector of gravitational torques and $\boldsymbol{\tau}_i \in \mathbb{R}^p$ is the force produced by the actuator of the i^{th} agent. Here, the leader specifies the objective for the follower network. The agents can measure relative positions using line of sight vector technique and a constant *unknown* bias, $\mathbf{b}_i \in \mathbb{R}^3$ for i^{th} agent, is present in these measurements. Now, we make the following assumptions:

Assumption 3.1. *All followers are connected to the leader and the communication network is undirected.*

Assumption 3.2. *Neighbors can exchange both, their measurement of relative position of the leader and their estimate of the bias.*

Assumption 3.3. *There exist positive constants k_{m_i}, k_{c_i} and k_g such that $M_i(q_i) - k_{m_i}\mathbf{I}_p \leq 0$, $||(g_i(q_i))|| \leq k_g$ and $||(C_i(q_i, \dot{q}_i))|| \leq k_{c_i}$*

Assumption 3.4. $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ *is skew symmetric*

The objective is for the followers to approach the generalized coordinates of the leader with local interaction. We propose a non linear, distributed and model independent adaptive control law which ensures asymptotic convergence to a neighborhood of the consensus. A Lyapunov based analysis is used to derive bias estimator dynamics.

3.3 Control Law Design

Define

$$s_i = \dot{\mathbf{q}}_i + \lambda(\mathbf{q}_i + \mathbf{b}_i - \hat{\mathbf{b}})_i, \quad \lambda \geq 0 \tag{3.7}$$

where $\mathbf{b}_i$ is the bias and $\hat{\mathbf{b}}_i$ is the estimate of the bias for the i^{th} agent. (3.2) can then be written as:

$$\mathbf{M}_i\dot{s}_i = \boldsymbol{\tau}_i - \mathbf{C}_i\dot{\mathbf{q}}_i - \mathbf{g}_i + \lambda\mathbf{M}_i(\dot{\mathbf{q}}_i - \dot{\hat{\mathbf{b}}}_i) \tag{3.8}$$

We propose the following control law :

$$\boldsymbol{\tau}_i = -\alpha \sum_{j=0}^{n} a_{ij}(s_i - s_j) - \beta_i sgn(s_i) - \gamma_i \left\|\dot{\mathbf{q}}_i\right\| sgn(s_i), \quad \alpha, \beta_i, \gamma_i \geq 0 \tag{3.9}$$

$$\boldsymbol{\tau} = -\alpha[(\mathcal{L} + \bar{A}) \otimes \mathbf{I}_3]s - \beta sgn(s) - \Gamma Q sgn(s) \tag{3.10}$$

where $\Gamma \triangleq blkdiag(\gamma_1 \mathbf{I}_3, ..., \gamma_n \mathbf{I}_3)$ and $Q \triangleq blkdiag(\|\dot{\mathbf{q}}\|_1 \mathbf{I}_3, ..., \|\dot{\mathbf{q}}\|_n \mathbf{I}_3)$ and $s \triangleq [s_1, ..., s_n]^T$. Define the following placeholders for brevity:

$$H = \mathcal{L} + \bar{A} \tag{3.11}$$

$$H_1 = H \otimes \mathbf{I}_3 \tag{3.12}$$

$$\tilde{\mathbf{b}} = \mathbf{b} - \hat{\mathbf{b}} \tag{3.13}$$

$$\tilde{\mathbf{q}} = \mathbf{q} + \tilde{\mathbf{b}} \tag{3.14}$$

The adaptive control law for estimating bias is taken to be:

$$\dot{\hat{\mathbf{b}}} = -\dot{\mathbf{q}} \tag{3.15}$$

Theorem 3.1. *Consider the multi-agent leader follower spacecraft network with agent dynamics given by* (3.6) *and an undirected connected communication graph* $\mathcal{G}$. *If Assumptions 1 - 4 hold, then the control law described by* (3.7)–(3.10) *and bias adaptation law* (3.15)*, guarantees that* $\lim_{t\to\infty} \dot{\boldsymbol{q}}(t) \to 0$, $\lim_{t\to\infty} [\boldsymbol{q}(t) + \tilde{\boldsymbol{b}}(t)] \to 0$ *exponentially.*

Proof. Consider the following Lyapunov function candidate

$$V = \frac{1}{2} s^T \mathbf{M} s \tag{3.16}$$

Taking derivative along dynamics and control from (3.6)–(3.10),

$$\begin{aligned} \dot{V} &= \frac{1}{2} s^T \dot{\mathbf{M}} s + s^T \mathbf{M} \dot{s} \\ &= s^T(-\alpha H_1 s - \beta \text{sgn}(s) - \Gamma Q \text{sgn}(s) - \mathbf{C}\dot{\mathbf{q}} - \mathbf{g} + \lambda \mathbf{M}(\dot{\mathbf{q}} - \dot{\hat{\mathbf{b}}})) \\ &= -\alpha s^T H_1 s - \beta \|s\|_1 - s^T \Gamma Q sgn(s) - s^T \mathbf{C}\dot{\mathbf{q}} - s^T \mathbf{g} + \lambda s^T \mathbf{M}(\dot{\mathbf{q}} - \dot{\hat{\mathbf{b}}}) \end{aligned} \tag{3.17}$$

Further, substituting (3.15) and using Lemmas 2 and 5, we have

$$\begin{aligned} \dot{V} &\leq -\alpha s^T H_1 s - \beta \|s\| + k_g \|s\| - s^T \Gamma Q \text{sgn}(s) - s^T \mathbf{C}\dot{\mathbf{q}} + 2\lambda s^T \mathbf{M}\dot{\mathbf{q}} \\ &\leq -\alpha s^T H_1 s - (\beta - k_g)\|s\| - \sum_{i=1}^{n} \gamma_i \left\|\dot{\mathbf{q}}_i\right\| \|s_i\|_1 - \sum_{i=1}^{n} s_i^T \mathbf{C}_i \dot{\mathbf{q}}_i + 2\lambda \sum_{i=1}^{n} s_i^T \mathbf{M}_i \dot{\mathbf{q}}_i \\ &\leq -\alpha s^T H_1 s - (\beta - k_g)\|s\| - \sum_{i=1}^{n} \gamma_i \left\|\dot{\mathbf{q}}_i\right\| \|s_i\| + \sum_{i=1}^{n} \|s_i\| \|\mathbf{C}_i\| \dot{\mathbf{q}}_i + 2\lambda \sum_{i=1}^{n} \|s_i\| \|\mathbf{M}_i\| \left\|\dot{\mathbf{q}}_i\right\| \\ &\leq -\alpha s^T H_1 s - (\beta - k_g)\|s\| + \sum_{i=1}^{n} (k_{c_i} + 2\lambda k_{m_i} - \gamma_i) \|s_i\| \left\|\dot{\mathbf{q}}_i\right\| \end{aligned} \tag{3.18}$$

If we choose

$$\beta > k_g \tag{3.19}$$

$$\gamma_i > k_{c_i} + 2\lambda k_{m_i} \tag{3.20}$$

We have

$$\begin{aligned}\dot{V} &\leq -\alpha s^T H_1 s \\ &\leq -\alpha\lambda_{min}(H)\,\|s\|^2 \end{aligned} \tag{3.21}$$

From (3.16) we have,

$$V \leq \frac{k_m}{2}\,\|s\|^2 \implies \|s\|^2 \geq \frac{2}{k_m}V$$

Substituting this in (3.21),

$$\dot{V} \leq -\eta V, \quad \eta = \frac{2\alpha\lambda_{min}(H)}{k_m} \geq 0$$

$$V(t) \leq V(0)e^{-\eta t} \tag{3.22}$$

(3.22) implies $\lim_{t\to\infty} V(t) \leq 0$. However, from (5.38) we have $V(t) \geq 0$ implying $\lim_{t\to\infty} V(t) = 0 \implies \lim_{t\to\infty} s(t) = 0$. Let the initial condition for position, velocity and bias be given by $\mathbf{q}(0), \dot{\mathbf{q}}(0)$ and $\tilde{\mathbf{b}}(0)$ respectively. Using (3.15) and the fact that $\lim_{t\to\infty} s(t) = 0$ we have,

$$\dot{\mathbf{q}} + \lambda(\mathbf{q} + \tilde{\mathbf{b}}) = 0 \tag{3.23}$$

Solving (3.15) and (3.23) using Laplace transform we get

$$\dot{\mathbf{q}} = -\lambda(\mathbf{q}(0) + \tilde{\mathbf{b}}(0))e^{-2\lambda t} \tag{3.24}$$

$$\mathbf{q} = \Big(\frac{\mathbf{q}(0) + \tilde{\mathbf{b}}(0)}{2}\Big)e^{-2\lambda t} + \Big(\frac{\mathbf{q}(0) - \tilde{b}(0)}{2}\Big) \tag{3.25}$$

$$\tilde{\mathbf{b}} = \Big(\frac{\mathbf{q}(0) + \tilde{\mathbf{b}}(0)}{2}\Big)e^{-2\lambda t} - \Big(\frac{\mathbf{q}(0) - \tilde{\mathbf{b}}(0)}{2}\Big) \tag{3.26}$$

Applying limit on (3.24), (3.25) and (3.26) to analyze the asymptotic behavior :

$$\lim_{t\to\infty} \dot{\mathbf{q}}(t) = 0 \tag{3.27}$$

$$\lim_{t\to\infty}[\mathbf{q}(t) + \tilde{\mathbf{b}}(t)] = 0 \tag{3.28}$$

$$\lim_{t\to\infty} \mathbf{q}(t) = \frac{\mathbf{q}(0) - \tilde{\mathbf{b}}(0)}{2} \tag{3.29}$$

$$\lim_{t\to\infty} \tilde{\mathbf{b}}(t) = -\Big(\frac{\mathbf{q}(0) - \tilde{\mathbf{b}}(0)}{2}\Big) \tag{3.30}$$

Hence, using the proposed control law we are able to achieve exponential convergence of velocity ($\dot{\mathbf{q}}$) and ($\mathbf{q} + \tilde{\mathbf{b}}$) as seen from (3.24)-(3.26) while position and bias converges to a constant value in the neighborhood of consensus. □

3.4 Simulations

In this section, simulations results are presented to validate our algorithm. We have considered one leader and five agents. All the agents are assumed to be connected to leader. The Laplacian and Adjacency matrix of the followers are given by:

$$\mathcal{L} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}, \qquad \mathcal{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{3.31}$$

The reference orbit (leader's orbit) is assumed to be circular with $r_l = 7078km$ and mass of each follower is identical, $m_i = 1kg$. The initial relative position of the followers is randomly chosen to lie between $[0, 9]m$ while the initial relative velocities lie in $[0, 6]m/s$. The true bias lies in the range of $[-1, 2]m$ for each coordinate of agents. The initial estimate of bias for i^{th} agent is initialized as, $\hat{\vec{b}} = (\vec{b}_i - 1)m$. The constants α, λ, β_i and γ_i are chosen to be 1, 0.5, 20.2 and 3.12 respectively.

Fig. (3.2) shows time variation of the compensated biased relative positions of the followers. It is evident by this figure that $\mathbf{q} + \tilde{\mathbf{b}} \to 0$ exponentially for all the agents. From Fig. (3.4) we can observe that the bounded trajectories are obtained for all the followers in the neighborhood of leader's orbit asymptotically. This bound on the trajectory depends on the initial value of the biased position. Fig. (3.3) shows the relative velocities of the followers with respect to leader. It can be seen that velocity for all the followers approach to that of leader exponentially.

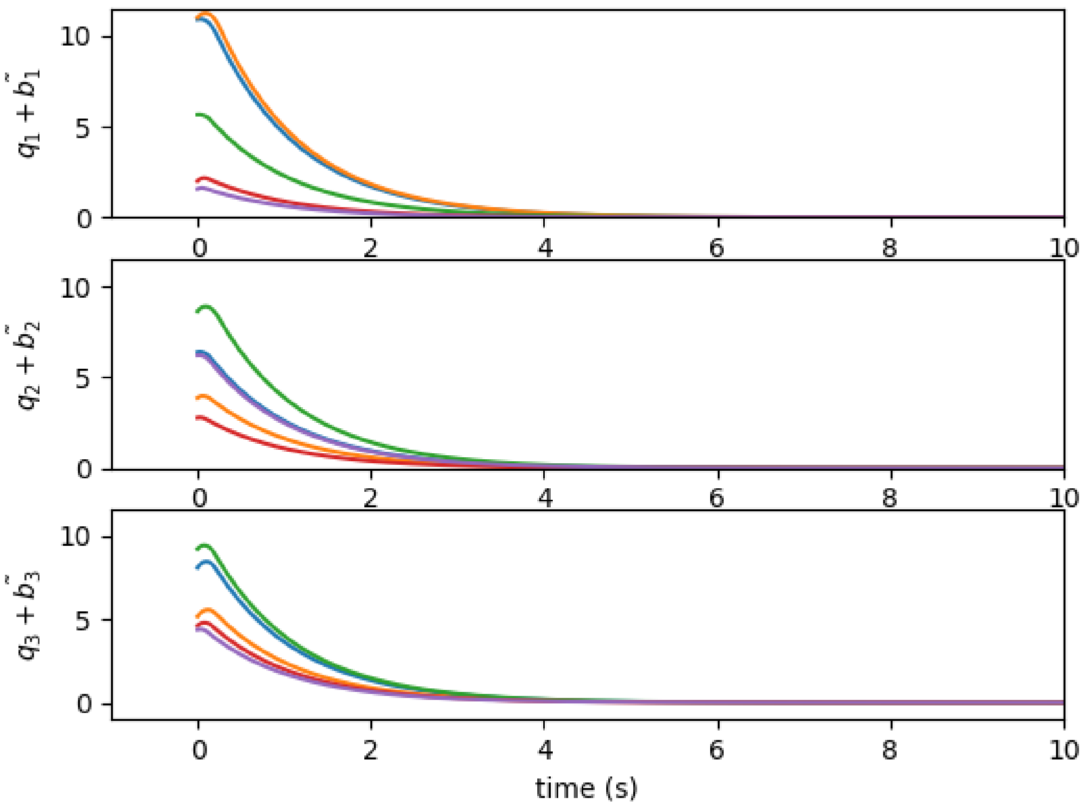

Figure 3.2: $[\mathbf{q}(t)+\tilde{\mathbf{b}}(t)](m)$ vs time (s). The sum of position and $\tilde{\mathbf{b}}$ exponentially converges to leader's trajectory

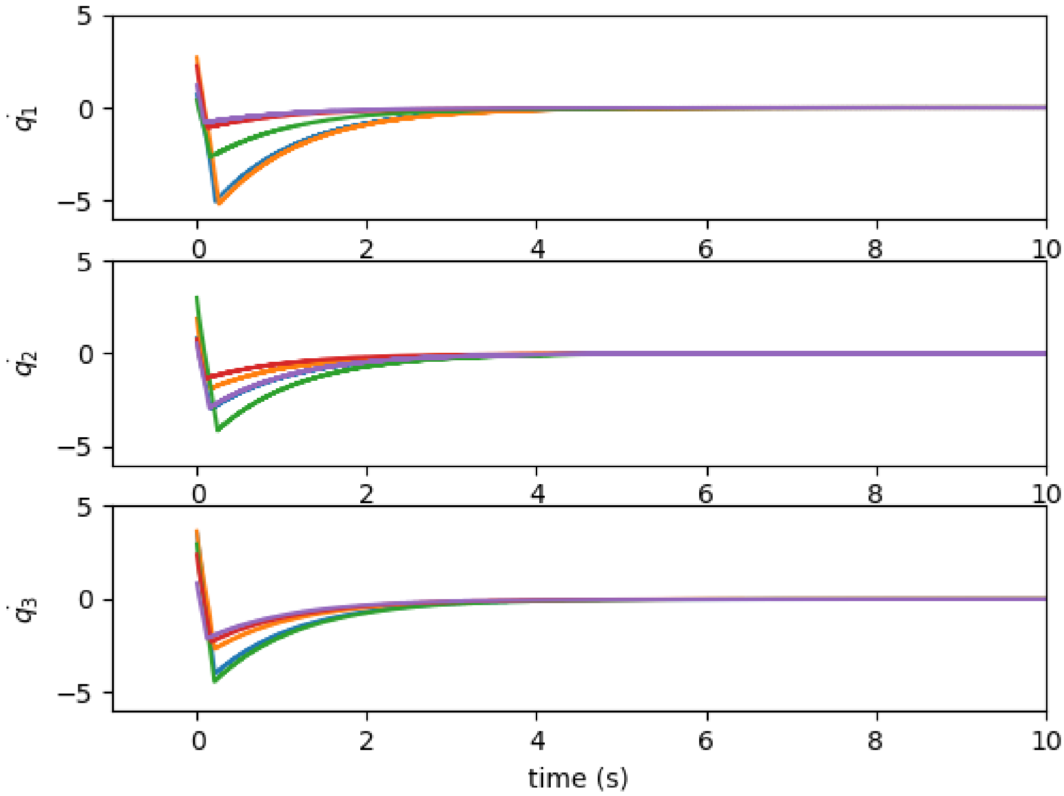

Figure 3.3: $\dot{\mathbf{q}}(t)(m/s)$ vs time (s). The velocity of all agents converges exponentially to the leader's velocity

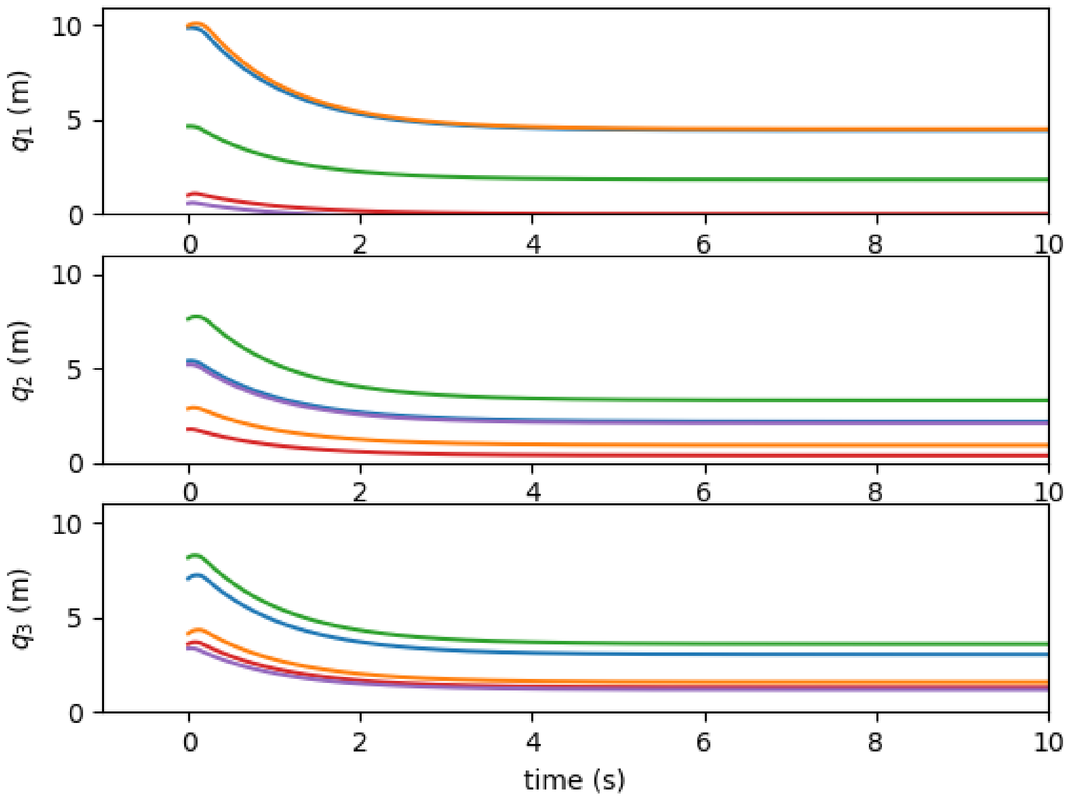

Figure 3.4: $\mathbf{q}(t)$ vs time. Position of the followers converges to a constant value in the neighborhood of the leader's trajectory

Chapter 4

Attempt 2 for Consensus Tracking

In the previous chapter an algorithm was developed that ensured exponential convergence of velocity and asymptotic convergence of position in the neighborhood of leader's trajectory. In this chapter we have attempted to reduce this error appreciably to ensure exponential convergence of position, velocity and estimated bias using a new form of composite adaptive controller. Here, the composite adaptive law is strategically designed to be proportional to the parameter estimation error in addition to the tracking error.

4.1 Control Law Design

The same problem of spacecraft synchronization as stated in previous chapter is considered here with assumptions (3.1) - (3.4) holding. Define,

$$s_i = \dot{\mathbf{q}}_i + \lambda(\mathbf{q}_i + \mathbf{b}_i - \hat{\mathbf{b}})_i, \quad \lambda \geq 0 \tag{4.1}$$

Substituting this in (3.2),

$$\dot{s}_i = \boldsymbol{\tau}_i - c_i \dot{\mathbf{q}}_i - \mathbf{g}_i + \lambda \dot{\mathbf{q}}_i - \dot{\hat{\mathbf{b}}}_i \tag{4.2}$$

where (4.2) is whole divided by the mass of the follower (which is constant and known) and then denoting new $c_i \triangleq \frac{\mathbf{C}_i}{\mathbf{M}_i}$ and new $\mathbf{g}_i \triangleq \frac{\mathbf{g}_i}{\mathbf{M}_i}$. Now, we propose the following control law:

$$\begin{aligned}\boldsymbol{\tau}_i &= -k_1 \sum_{j=0}^{n} a_{ij}(\mathbf{q}_i - \mathbf{q}_j + \mathbf{b}_i - \hat{\mathbf{b}}_i) - k_s \sum_{j=0}^{n} a_{ij}(s_i - s_j) - \beta_i \operatorname{sgn}(s_i) \\ &\quad - \gamma_i \left\|\dot{\mathbf{q}}_i\right\| - \lambda \dot{\mathbf{q}}_i + \dot{\hat{\mathbf{b}}}_i, \qquad \alpha, \beta_i, \gamma_i \geq 0 \end{aligned} \tag{4.3}$$

$$\begin{aligned}\boldsymbol{\tau} &= -k_1[\mathcal{L} + \bar{A} \otimes I_3]\mathbf{q} - k_1[\mathcal{D} + \bar{A} \otimes I_3](\mathbf{b} - \hat{\mathbf{b}}) - k_s[\mathcal{L} + \bar{A} \otimes I_3]s \\ &\quad - \beta \operatorname{sgn}(s) - \Gamma Q \operatorname{sgn}(s) - \lambda \dot{\mathbf{q}} + \dot{\hat{\mathbf{b}}}\end{aligned} \tag{4.4}$$

where $\Gamma \triangleq \text{blkdiag}(\gamma_1 I_3, \cdots \gamma_n I_3)$, $Q \triangleq \text{blkdiag}\Big(\|\dot{\mathbf{q}}_1\|, \cdots, \|\dot{\mathbf{q}}_n\|\Big)$ and $s \triangleq [s_1, \cdots s_n]^T$. Let $D_1 \triangleq \mathcal{D} + \bar{A}$ and other placeholders remain same as defined in Chapter 3,

$$\dot{s} = -k_1 H_1 \mathbf{q} - k_1 D_1 \tilde{\mathbf{b}} - k_s H_1 s - \beta \text{sgn}(s) - \Gamma Q \text{sgn}(s) - c\dot{\mathbf{q}} - \mathbf{g} \tag{4.5}$$

The adaptive control is then taken to be,

$$\dot{\tilde{\mathbf{b}}} = D_1 \dot{\mathbf{q}} + \lambda(H_1 + D_1)(\mathbf{q}) \tag{4.6}$$

(4.6) cannot be implemented due to the presence of $\mathbf{q} - \tilde{\mathbf{b}}$, which in non-measurable. To obviate the need of $\mathbf{q} - \tilde{\mathbf{b}}$, following procedure can be applied : Let $\dot{\tilde{\mathbf{b}}} = r$ and $\lambda(D_1 + H_1) = k$, then

$$r = D_1 \dot{\mathbf{q}} + k(\mathbf{q} - \tilde{\mathbf{b}}) \tag{4.7}$$

$$\dot{r} = D_1 \ddot{\mathbf{q}} + k(\dot{\mathbf{q}} - r) \tag{4.8}$$

$$\frac{d(e^{kt} r)}{dt} = k\dot{\mathbf{q}} + D_1 \ddot{\mathbf{q}} \tag{4.9}$$

$$r(t) = e^{-kt} r(0) + e^{-kt} \int_0^t e^{ku} \dot{\mathbf{q}}(u) du + e^{-kt} \int_0^t e^{kt} D_1 \ddot{\mathbf{q}}(u) du \tag{4.10}$$

The last part of the above equation can be integrated using by parts;

$$r(t) = e^{kt} r(0) + e^{-kt} \int_0^t e^{ku} k\dot{\mathbf{q}}(u) du + e^{-kt} [e^{ku} D_1 \dot{\mathbf{q}}(u)]_0^t - e^{-kt} \int_0^t k e^{ku} D_1 \dot{\mathbf{q}}(u) du \tag{4.11}$$

$$r(t) = \dot{\tilde{\mathbf{b}}}(t) = e^{-kt} \dot{\tilde{\mathbf{b}}}(0) + e^{-kt} h_1(t) + D_1 \dot{\mathbf{q}}(t) - e^{-kt} D_1 \dot{\mathbf{q}}(0) - e^{-kt} h_2(t) \tag{4.12}$$

$$\dot{h}_1 = k e^{kt} \dot{\mathbf{q}}(t), \qquad h_1(0) = 0 \tag{4.13}$$

$$\dot{h}_2 = k e^{kt} D_1 \dot{\mathbf{q}}(t), \qquad h_2(0) = 0 \tag{4.14}$$

Using (4.12), (4.13) and (4.14), $\dot{\hat{\mathbf{b}}}$ can be computed online. But the initial condition for $\dot{\tilde{\mathbf{b}}}$ is still unknown and hence the bias convergence depends on how precisely the initial condition is specified.

4.2 Lyapunov Analysis

Consider the following Lyapunov function candidate,

$$V = \frac{1}{2} s^T s + \frac{1}{2} \mathbf{q}^T H_1 \mathbf{q} + \frac{1}{2} \tilde{\mathbf{b}}^T \tilde{\mathbf{b}} \tag{4.15}$$

Let $\zeta(t) = [\mathbf{q}(t)^T \; \tilde{\mathbf{b}}^T \; s^T]^T$. Then,

$$V(t) \leq c_1 \|\zeta(t)\|^2, \qquad c_1 = max\Big[\frac{1}{2} \lambda_{max}(H) \frac{k_1}{2} \frac{1}{2}\Big] \tag{4.16}$$

Taking derivative of (4.15) along dynamics and control and substituting (4.5) and (4.6),

$$\begin{aligned}
\dot{V} &= s^T s + k_1 \mathbf{q}^T H_1 \dot{\mathbf{q}} + \tilde{\mathbf{b}}^T \dot{\tilde{\mathbf{b}}} \\
&= s^T(-k_1 H_1 \mathbf{q} - k_1 D_1 \tilde{\mathbf{b}} - k_s H_1 s - \beta \mathrm{sgn}(s) - \Gamma Q \mathrm{sgn}(s) - c\dot{\mathbf{q}} - \mathbf{g}) \\
&+ k_1 \mathbf{q}^T H_1 (s - \lambda \mathbf{q} - \lambda \tilde{\mathbf{b}}) + k_1 \tilde{\mathbf{b}}^T \dot{\tilde{\mathbf{b}}} \\
&= -k_s s^T H_1 s - k_1 \lambda \mathbf{q}^T H_1 \mathbf{q} - \beta \|s\|_1 - s^T \mathbf{g} - s^T \Gamma \mathrm{sgn}(s) - c\mathbf{q} \dot{-} \mathbf{g} \\
&+ k_1 \tilde{\mathbf{b}}^T [\dot{\tilde{\mathbf{b}}} - \lambda H_1 \mathbf{q} - D_1 s] \\
&\leq -k_s s^T H_1 s - k_1 \lambda \mathbf{q}^T H_1 \mathbf{q} - (\beta - k_g) \|s\| - \sum_{i=1}^{n} (k_{c_i} - \gamma_i) \|s_i\| \left\| \dot{\mathbf{q}}_i \right\| \\
&+ k_1 \tilde{\mathbf{b}}^T [\dot{\tilde{\mathbf{b}}} - D_1 \dot{\mathbf{q}} - \lambda (H_1 + D_1) \mathbf{q} - \lambda D_1 \tilde{\mathbf{b}}]
\end{aligned} \tag{4.17}$$

Substituting the adaption law (4.6) in the above equation and choosing;

$$\beta > k_g \tag{4.18}$$

$$\gamma_i > k_{c_i} \tag{4.19}$$

We have,

$$\begin{aligned}
\dot{V} &\leq -k_s s^T H_1 s - k_1 \lambda \mathbf{q}^T H_1 \mathbf{q} - k_1 \lambda (H_1 + 2D_1) \tilde{b} \\
&\leq k_1 \lambda \lambda_{min}(H) \|s\|^2 - k_1 \lambda \lambda_{min}(H) \|\mathbf{q}\|^2 - k_1 \lambda [\lambda_{min}(H) + 2\lambda_{min}(D)] \left\| \tilde{\mathbf{b}} \right\|^2 \\
&\leq -c_2 \|\zeta(t)\|^2, \qquad \min[\lambda_{min}(H, \lambda_{min}(H) + 2\lambda_{min}(D)]
\end{aligned} \tag{4.20}$$

Substituting (4.17) in (4.20),

$$\dot{V}(t) \leq -\frac{c_2}{c_1} V(t) \tag{4.21}$$

$$V(t) \leq V(0) e^{-\frac{c_2}{c_1} t} \tag{4.22}$$

This implies that $\zeta(t)$ is global stable exponential function, that is,

$$\lim_{t \to \infty} \zeta(t) = 0 \tag{4.23}$$

$$\lim_{t \to \infty} \mathbf{q}(t) = 0, \qquad \lim_{t \to \infty} \tilde{\mathbf{b}}(t) = 0, \qquad \lim_{t \to \infty} \dot{\mathbf{q}}(t) = 0 \tag{4.24}$$

But as stated earlier we do not exactly know the initial condition for the $\tilde{\mathbf{b}}$ and hence all the followers converges to a neighborhood of leaderâĂŹs trajectory exponentially with error in convergence depending on how precise initial condition for adaptive controller is chosen.

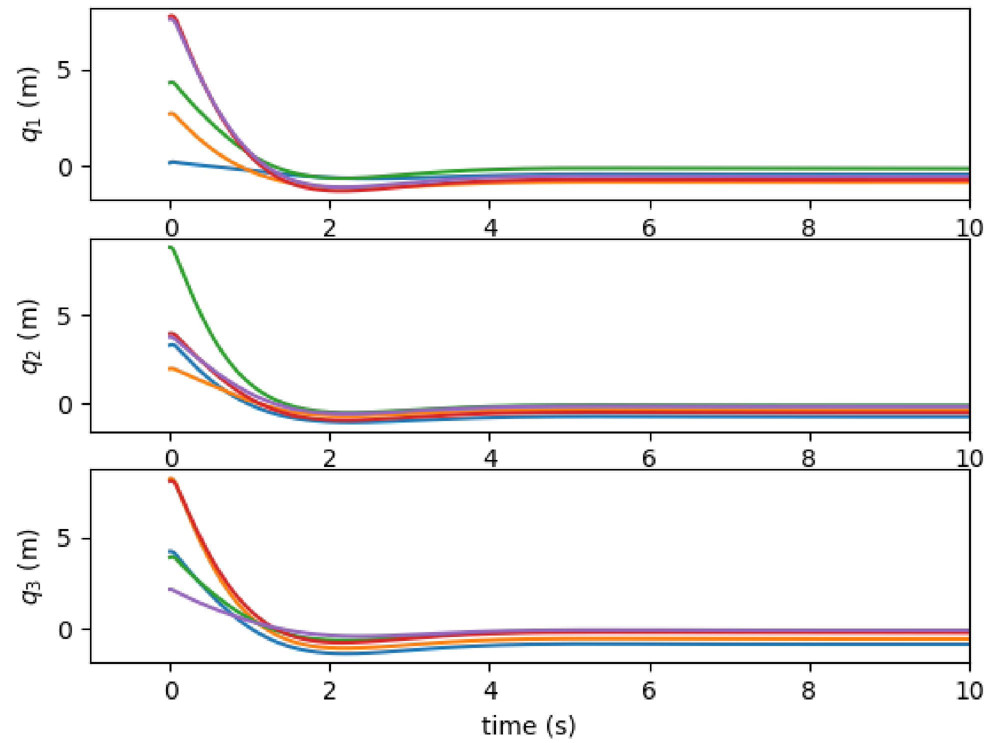

Figure 4.1: $\mathbf{q}(t)(m)$ vs time (s).

4.3 Simulations

We have considered the similar setup as in previous chapter with the control law defined by (4.4) and (4.6). The initail condition for (4.12) is chosen to be:

$$\dot{\tilde{\mathbf{b}}} = D_1\dot{\mathbf{q}}(0) + \lambda(H_1 + D_1)(\mathbf{q}(0 + \tilde{\mathbf{b}}(0)) \tag{4.25}$$

The consensus error is given by $2\tilde{\mathbf{b}}(0)$(subtract (4.25) from (4.6)). This error can be reduced if the initial estimate of the bias is close enough to the actual bias. Figure (4.1) show the time variation of the relative positions of the followers. It is evident by this figure that the relative position converges exponentially in the close neighborhood of the leader, for all the agents. This bound on the trajectory depends on the accuracy of the initialization of $\dot{\tilde{\mathbf{b}}}$. Figure (4.3) show the relative velocities of the followers with respect to leader. It can be seen that velocity for all the followers approach to that of leader exponentially. Bias estimation also converges in the neighborhood of the actual system bias exponentially as evident from Figure (4.2). The results obtained for this algorithm are better than the previous since the consensus error has reduced significantly.

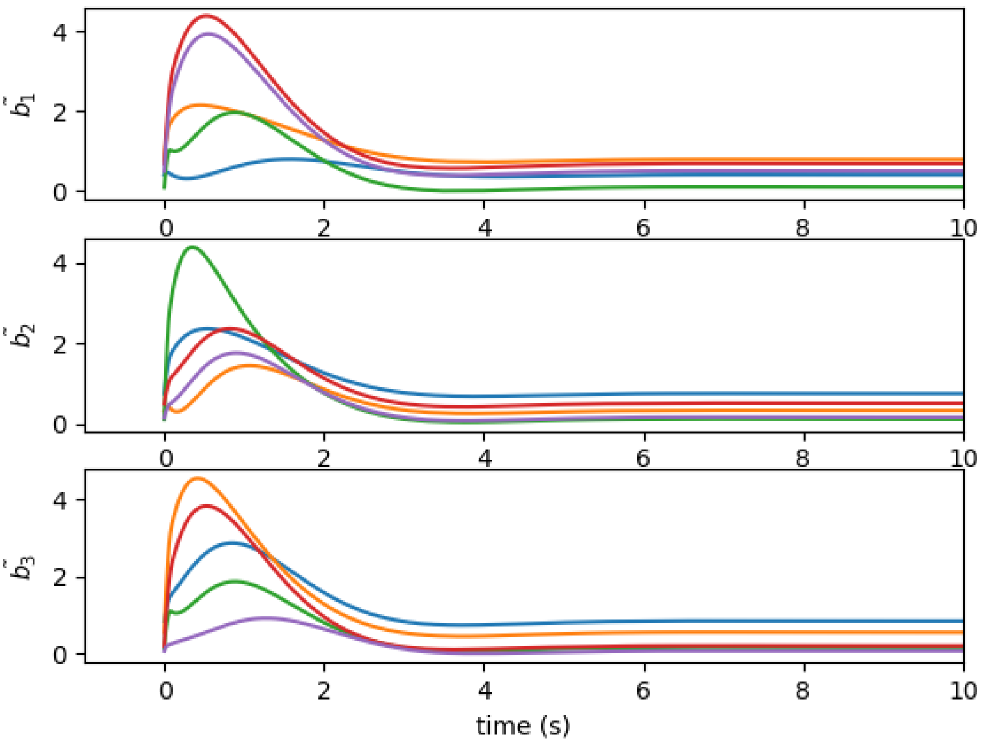

Figure 4.2: $\tilde{\mathbf{b}}(t)(m)$ vs time (s).

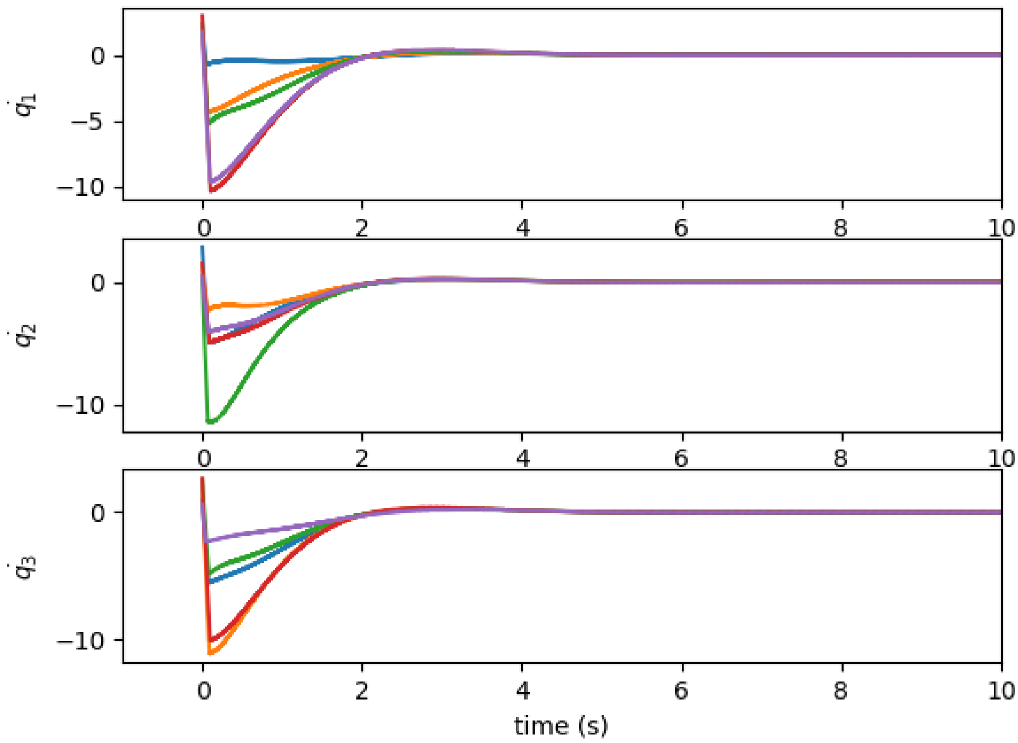

Figure 4.3: $\dot{\mathbf{q}}(t)(m)$ vs time (s).

Chapter 5

Final Algorithm for Consensus Tracking

In this chapter we develop an algorithm which ensures bias estimation errors are exponentially converged to zero without any knowledge of upper bound on the bias while also achieving asymptotic convergence of both position and velocity to that of leader's trajectory. Simulation results are presented for a five two-link revolute joint arms(followers) tracking a leader with local interaction only.

5.1 Model Description and Control Objective

The objective of this chapter is to develop a distributed consensus tracking algorithm for a multi agent system in the presence of a constant unknown bias in the relative measurement of position, while ensuring estimation of the bias. The dynamics for agents in $\mathcal{G}$ is given by a subclass of Euler-Lagrangian (EL) equation,

$$M_i(\mathbf{q})_i\ddot{\mathbf{q}}_i + C_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + F_i(\dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i = \boldsymbol{\tau}_i \tag{5.1}$$

Here, m is the dimension of generalized coordinates. (5.1) is a subclass of EL equation since the gravitational force term, $\mathbf{g}_i(\mathbf{q}_i)$, is not included in it. The tracking error for i^{th} agent is given by,

$$\mathbf{e}_i(t) = \mathbf{q}_i - \mathbf{q}_0 \tag{5.2}$$

The consensus objective is achieved if,

$$\lim_{t\to\infty} \mathbf{e}_i = \mathbf{0}, \qquad \forall i = 1\cdots n \tag{5.3}$$

$$\lim_{t\to\infty} \dot{\mathbf{e}}_i = \mathbf{0}, \qquad \forall i = 1\cdots n \tag{5.4}$$

$$\lim_{t\to\infty} \mathbf{b}_i - \hat{\mathbf{b}}_i = \mathbf{0}, \qquad \forall i = 1\cdots n \tag{5.5}$$

5.2 Control Law Design

In this section we develop a controller $\boldsymbol{\tau}(t)$ which tracks the leader's trajectory $\mathbf{q}_0$ in the presence of unknown bias in the relative position measurements. Below are few assumption we make,

Assumption 5.1. *The network graph, $\mathcal{G}$, is undirected and connected*

Assumption 5.2. *Atleast one agent is connected to the leader*

Assumption 5.3. *Agents can measure their own velocity $\dot{\boldsymbol{q}}$ and neighbors measure the relative position of each other in the presence of a constant unknown bias*

Assumption 5.4. *Neighbors can exchange their measurement of relative position of each other*

Assumption 5.5. *The trajectory of the leader $\boldsymbol{q}_0 \in C^2$ functions and $\boldsymbol{q}_0, \dot{\boldsymbol{q}}_0, \ddot{\boldsymbol{q}}_0 \in \mathcal{L}_\infty$. Furthermore, it is assumed that $\dot{\boldsymbol{q}}_0$ and $\ddot{\boldsymbol{q}}_0$ are known to the followers.*

Assumption 5.6. *The matrices M_i, C_i and F_i are known to the agent.*

Now, rewriting (5.1),

$$\ddot{\mathbf{q}}_i = M_i^{-1}(\boldsymbol{\tau}_i - C_i\dot{\mathbf{q}}_i - F_i\dot{\mathbf{q}}_i) \tag{5.6}$$

Remark 1. *The Assumption* (5.6) *can be omitted if the dynamics of the agents in the network is simplified to double integrator system*

Consider the following control algorithm,

$$\begin{aligned}
\boldsymbol{\tau}_i = &-k_1(t)\sum_{j=0}^{n} a_{ij}(\dot{\mathbf{q}}_i - \dot{\mathbf{q}}_j) + \frac{k_1(t)}{2}\sum_{j=0}^{n} a_{ij}[(\mathbf{q}_i - \mathbf{q}_j + \mathbf{b}_i) - (\mathbf{q}_j - \mathbf{q}_i + \mathbf{b}_j)] \\
&- k_1(t)\sum_{j=0}^{n} a_{ij}(\mathbf{q}_i - \mathbf{q}_j + \mathbf{b}_i) + u_i' + C_i\dot{\mathbf{q}}_i + F_i\dot{\mathbf{q}}_i \\
= &-k_1(t)\sum_{j=0}^{n} a_{ij}(\dot{\mathbf{q}}_i - \dot{\mathbf{q}}_j) + k_1(t)\cancel{\sum_{j=0}^{n} a_{ij}(\mathbf{q}_i - \mathbf{q}_j)} + \frac{k_1(t)}{2}\sum_{j=0}^{n} a_{ij}(\mathbf{b}_i - \mathbf{b}_j) \\
&- k_1(t)\cancel{\sum_{j=0}^{n} a_{ij}(\mathbf{q}_i - \mathbf{q}_j)} - k_1(t)\sum_{j=0}^{n} a_{ij}\,\mathbf{b}_i + \mathbf{u}_i' + C_i\dot{\mathbf{q}}_i + F_i\dot{\mathbf{q}}_i
\end{aligned} \tag{5.7}$$

$$\boldsymbol{\tau} = -k_1(t)[(\mathcal{L} + \bar{A}) \otimes I_m]\dot{\mathbf{e}} + \frac{k_1(t)}{2}[(\mathcal{L} + \bar{A}) \otimes I_n]\mathbf{b} - k_1(t)[(\mathcal{D} + \bar{A}) \otimes I_m]\mathbf{b} + \mathbf{u}' + C\dot{\mathbf{q}} + F\dot{\mathbf{q}} \tag{5.8}$$

where, $k_1(t)$ is a scalar function of time and $\boldsymbol{\tau}$, $\mathbf{e}$, $\mathbf{b}$ and $\mathbf{u}'$ are the column stack vector of $[\tau_1 \cdots \tau_n]^T$, $[e_1 \cdots e_n]^T$, $[b_1 \cdots b_n]^T$ and $[u'_1 \cdots u'_n]^T$ respectively. Let $\bar{L} = (\mathcal{L} + \bar{A} \otimes I_m)$ and $\bar{D} = (\mathcal{D} + \bar{A}) \otimes I_n$, then (5.8) can be written as,

$$\boldsymbol{\tau} = -k_1(t)\bar{D}\mathbf{b} - k_1(t)\bar{L}\dot{\mathbf{e}} + \frac{k_1(t)}{2}\bar{L}\mathbf{b} + \mathbf{u}' + C\dot{\mathbf{q}} + F\dot{\mathbf{q}} \tag{5.9}$$

Rewriting (5.6) and substituting (5.9),

$$\ddot{\mathbf{q}} = M^{-1}\Big(- k_1(t)\bar{D}\mathbf{b} - k_1(t)\bar{L}\dot{\mathbf{e}} + \frac{k_1(t)}{2}\bar{L}\mathbf{b} + \mathbf{u}' + \cancel{C\dot{\mathbf{q}}} + \cancel{F\dot{\mathbf{q}}} - \cancel{C\dot{\mathbf{q}}} - \cancel{F\dot{\mathbf{q}}}\Big) \tag{5.10}$$

$$\ddot{\mathbf{q}} + M^{-1}k_1(t)\bar{L}\dot{\mathbf{e}} + M^{-1}k_1(t)\Big(\bar{D} - \frac{1}{2}\bar{L}\Big)\mathbf{b} = \mathbf{u} \tag{5.11}$$

where $M = \text{blkdiag}(M_1 \cdots M_n)$ and $\mathbf{u} = M^{-1}\mathbf{u}'$. (5.10) can be written in a matrix form as,

$$\underbrace{\Big[\ddot{\mathbf{q}} \quad M^{-1}k_1(t)\bar{L}\dot{\mathbf{e}} \quad M^{-1}k_1(t)\Big(\bar{D} - \tfrac{1}{2}\bar{L}\Big)\Big]}_{Y}\underbrace{\Big[1 \quad 1 \quad \mathbf{b}\Big]^T}_{\theta} = \mathbf{u} \tag{5.12}$$

Define,

$$s_i = \dot{\mathbf{q}}_i + \lambda\Big(\mathbf{q}_i + \frac{1}{2}\tilde{\mathbf{b}}_i\Big), \qquad \lambda \in \mathbb{R}^+ \tag{5.13}$$

where $\tilde{\mathbf{b}}_i = \mathbf{b}_i - \hat{\mathbf{b}}_i$. Now, stacking all s_i and $\mathbf{e}_i$ in column vectors, let us define $\tilde{s} = s - s_0$ as,

$$\tilde{s} = \dot{\mathbf{e}} + \lambda\Big(\mathbf{e} + \frac{\tilde{\mathbf{b}}}{2}\Big) \tag{5.14}$$

Taking the derivative of(5.14) along the dynamics,

$$\begin{aligned}\dot{\tilde{s}} &= \ddot{\mathbf{q}} - \ddot{\mathbf{q}}_0 + \lambda\Big(\dot{\mathbf{q}} - \dot{\mathbf{q}}_0 + \frac{\dot{\tilde{\mathbf{b}}}}{2}\Big)\\ &= -M^{-1}k_1(t)\bar{L}\dot{\mathbf{e}} - M^{-1}k_1(t)\Big(\bar{D} - \frac{\bar{L}}{2}\Big) + \mathbf{u} - \ddot{\mathbf{q}}_0 + \lambda\Big(\dot{\mathbf{q}} - \dot{\mathbf{q}}_0 + \frac{\dot{\tilde{\mathbf{b}}}}{2}\Big) \qquad (5.15)\\ &= \underbrace{\Big[\mathbf{0}_{mn\times 1} \quad -M^{-1}k_1(t)\bar{L}\dot{\mathbf{e}} \quad -M^{-1}k_1(t)\Big(\bar{D} - \tfrac{\bar{L}}{2}\Big)\Big]}_{Z}\Big[1 \quad 1 \quad \mathbf{b}\Big]^T + \mathbf{u} - \ddot{\mathbf{q}}_0 + \lambda\Big(\dot{\mathbf{q}} - \dot{\mathbf{q}}_0 + \frac{\dot{\tilde{\mathbf{b}}}}{2}\Big)\end{aligned} \tag{5.16}$$

Let the second part of control algorithm, $\mathbf{u}$, be given by,

$$\mathbf{u} = -Z\hat{\theta} - k_2\bar{L}\tilde{s} - \lambda\dot{\mathbf{q}} - \frac{\lambda\dot{\tilde{\mathbf{b}}}}{2} + \ddot{\mathbf{q}}_0 + \lambda\dot{\mathbf{q}}_0, \qquad k_2 \in \mathbb{R}^+ \tag{5.17}$$

where $\hat{\theta}$ is the estimation of θ and $k_2 \in \mathbb{R}^+$. Substituting (5.17) in (5.16),

$$\dot{\tilde{s}} = Z\tilde{\theta} - k_2\bar{L}\tilde{s} \tag{5.18}$$

where $\tilde{\theta} = \theta - \hat{\theta}$.

5.2.1 Bias Estimation

The matrix $Y(\dot{\mathbf{q}}, \ddot{\mathbf{q}})$ is dependent on the acceleration term $\ddot{\mathbf{q}}(t)$ and so cannot be used in our adaption law for bias estimation. To obviate the need of $\ddot{\mathbf{q}}(t)$ a filter is designed as follows,

$$\dot{Y}_F = -kY_F + Y, \qquad Y_F(0) = 0 \tag{5.19}$$

$$\dot{\mathbf{u}}_F = -k\mathbf{u}_F + \mathbf{u}, \qquad \mathbf{u}_F(0) = 0 \tag{5.20}$$

where $k \in \mathbb{R}^+$ is the scalar filter gain, $Y_F(t) \in \mathbb{R}^{mn\times(2mn+1)}$ and $\mathbf{u}_F \in \mathbb{R}^{mn}$. Solving above equations explicitly,

$$Y_F(t) = e^{-kt}\int_0^t e^{k\sigma}Y(\sigma)d\sigma \tag{5.21}$$

$$\mathbf{u}_F(t) = e^{-kt}\int_0^t e^{k\sigma}\mathbf{u}(\sigma)d\sigma \tag{5.22}$$

Using the relation $Y\theta = \mathbf{u}$ we get $Y_F\theta = \mathbf{u}_F$ from (5.21) and (5.22). (5.19) cannot be solved explicitly as $Y(\dot{\mathbf{q}}, \ddot{\mathbf{q}})$ is not measurable. Therefore, splitting $Y(\dot{\mathbf{q}}, \ddot{\mathbf{q}})$ into measurable and non-measurable parts as,

$$Y(\dot{\mathbf{q}}, \ddot{\mathbf{q}}) = Y_1(\dot{\mathbf{q}}) + Y_2(\ddot{\mathbf{q}}) \tag{5.23}$$

where

$$Y_1 = \begin{bmatrix}\ddot{\mathbf{q}} & \mathbf{0}_{mn\times1} & \mathbf{0}_{mn\times1}\end{bmatrix} \tag{5.24}$$

$$Y_2 = \begin{bmatrix}\mathbf{0}_{mn\times1} & M^{-1}k_1(t)\bar{L}\dot{\mathbf{e}} & M^{-1}k_1(t)\left(\bar{D} - \frac{\bar{L}}{2}\right)\end{bmatrix} \tag{5.25}$$

Then $Y_F = Y_{F_1} + Y_{F_2}$, where,

$$\dot{Y}_{F_1} = -kY_{F_1} + Y_1, \qquad Y_{F_1}(0) = 0 \tag{5.26}$$

$$\dot{Y}_{F_2} = -kY_{F_2} + Y_2, \qquad Y_{F_2}(0) = 0 \tag{5.27}$$

Since Y_2 is known, Y_{F_2} can be solved online using (5.27) and for Y_{F_1}, solving it explicitly,

$$\begin{aligned} Y_{F_1} &= e^{-kt}\int_0^t e^{kx}Y_1(x)dx \\ &= \begin{bmatrix} \vdots & \vdots & \vdots \\ e^{-kt}\int_0^t e^{kx}\ddot{\mathbf{q}}_i^j dx & 0 & 0 \\ \vdots & \vdots & \vdots \end{bmatrix} \end{aligned} \tag{5.28}$$

The elements of Y_{F_1} can be obtained using integration by parts as follows,

$$Y_{F_1}(t) = \begin{bmatrix} \vdots & \vdots & \vdots \\ e^{-kt}[e^{kt}\dot{\mathbf{q}}_i^j(t) - \dot{\mathbf{q}}_i^j(0)] - e^{-kt}\int_0^t ke^{kx}\dot{\mathbf{q}}_i^j dx & 0 & 0 \\ \vdots & \vdots & \vdots \end{bmatrix}$$

$$= \begin{bmatrix} \vdots & \vdots & \vdots \\ \dot{\mathbf{q}}_i^j(t) - e^{-kt}\dot{\mathbf{q}}_i^j(0) - h_i^j(t) & 0 & 0 \\ \vdots & \vdots & \vdots \end{bmatrix} \tag{5.29}$$

where $j = 1, \cdots, m \;\forall\; i = 1, \cdots n$ and

$$\dot{h}_i^j(t) = k\dot{\mathbf{q}}_i^j - kh_i^j, \qquad h_i^j(0) = 0 \tag{5.30}$$

Y_F can now be used in our adaption law. Additionally, for bias estimation we will make use of the projection based integral law introduced by [16] as,

$$\begin{aligned} \dot{Y}_{IF} &= proj(Y_F^T Y_F), \qquad Y_{IF}(0) = 0 \\ &= \left\{ \begin{array}{ll} 0 & \text{if } (\|y_{IF}\|^2 \geq \text{L}) \text{ and } (y_F^T y_F) > 0 \\ Y_F^T Y_F & \text{otherwise} \end{array} \right\} \end{aligned} \tag{5.31}$$

$$\begin{aligned} \dot{\mathbf{u}}_{IF} &= proj(Y_F^T \mathbf{u}_F), \qquad \mathbf{u}_{IF}(0) = 0 \\ &= \left\{ \begin{array}{ll} 0 & \text{if } (\|y_{IF}\|^2 \geq \text{L}) \text{ and } (y_F^T y_F) > 0 \\ Y_F^T \mathbf{u}_F & \text{otherwise} \end{array} \right\} \end{aligned} \tag{5.32}$$

where $y_F = [vec^T(Y_F^T Y_F) \;\; \mathbf{u}_F^T Y_F]^T$, $Y_{IF} = [vec^T(Y_{IF}) \;\; \mathbf{u}_{IF}^T]^T$, $vec(A)$ denotes the vectorization of a matrix A and $\text{L} \in \mathbb{R}^+$ is the safety limit of the controller. Below are the properties of the projection operator. For the detailed description refer to [16].

Property 1. *[16] Integrating* (5.31) *and* (5.32) *and using the relation $Y_F\theta = \boldsymbol{u}_F$ it can be shown that,*

$$Y_{IF}\theta(t) = \boldsymbol{u}_{IF}, \qquad \forall t \geq 0 \tag{5.33}$$

Property 2. *[16] $Y_{IF}(t)$ is a positive semi-definite function of time i.e. $Y_{IF}(t) \geq 0, \quad \forall t \geq 0$*

Property 3. *[16] $Y_{IF}(t)$ is a non-decreasing function of time in the sense of matrix inequality i.e. $Y_{IF}(t_2) \geq Y_{IF}(t_1) \;\forall\; t_2 \geq t_1$*

The adaptive control law for bias estimation is now taken to be.

$$\dot{\hat{\theta}} = Z^T \bar{L}\tilde{s} + k_3(\mathbf{u}_{IF} - Y_{IF}\hat{\theta}) \tag{5.34}$$

$$\dot{\hat{\mathbf{b}}} = \dot{\hat{\theta}}(3 : end) \tag{5.35}$$

where $k_3 \in \mathbb{R}^+$. Further consider the following assumption,

Assumption 5.7. *[16] There exists T, $\gamma > 0$ such that $Y_F(t)$ is initially exciting (IE) during the interval [0,T], with the degree of excitation γ, i.e.*

$$\int_0^T Y_F^T(t)Y_F(t)dt \geq \gamma I_{mn} \tag{5.36}$$

5.3 Proposed Theorem for Consensus Tracking

Theorem 5.1. *Consider the multi-agent leader-follower network with agent dynamics given by* (5.1)*. If assumptions* (5.1) - (5.7) *hold, then the control law given by,*

$$\boldsymbol{\tau} = -k_1(t)\bar{D}\boldsymbol{b} - k_1(t)\bar{L}\dot{\boldsymbol{q}} + \frac{k_1(t)}{2}\bar{L}\boldsymbol{b} + C\dot{\boldsymbol{q}} + F\dot{\boldsymbol{q}} + M(-Z\hat{\theta} - k_2\bar{L}\tilde{s} - \lambda\dot{\boldsymbol{q}} - \frac{\lambda\dot{\hat{\boldsymbol{b}}}}{2} + \ddot{\boldsymbol{q}}_0 + \lambda\dot{\boldsymbol{q}}_0) \tag{5.37}$$

with bias adaption law (5.34)*, guarantees that* $\lim_{t\to\infty}\boldsymbol{e}(t) = 0$*,* $\lim_{t\to\infty}\dot{\boldsymbol{e}}(t) = 0$ *and* $\lim_{t\to\infty}(\boldsymbol{b}-\hat{\boldsymbol{b}}) = 0$ *exponentially.*

Proof. Consider the following Lyapunov candidate,

$$V = \frac{1}{2}\tilde{s}^T\bar{L}\tilde{s} + \frac{1}{2}\tilde{\theta}^T\tilde{\theta} \tag{5.38}$$

Taking derivative along dynamics,

$$\dot{V} = \tilde{s}^T\dot{\tilde{s}} + \tilde{\theta}^T\dot{\tilde{\theta}} \tag{5.39}$$

$$\dot{\tilde{\theta}} = -\dot{\hat{\theta}} \tag{5.40}$$

Substituting (5.16) and (5.34) in (5.39),

$$\dot{V} = \tilde{s}^T\bar{L}\Big(Z\tilde{\theta} - k_2\bar{L}\tilde{s}\Big) + \tilde{\theta}^T\Big(- Z^T\bar{L}\tilde{s} - k_3(\mathbf{u}_{IF} - Y_{IF}\hat{\theta})\Big) \tag{5.41}$$

Using (5.33),

$$\begin{aligned}\dot{V} &= -k_2\tilde{s}^T\bar{L}^2\tilde{s} - k_3\tilde{\theta}^TY_{IF}\tilde{\theta} \\ &\leq -k_2\lambda_{min}(\bar{L}^2)\,||\tilde{s}||^2 - k_3\gamma\left\|\tilde{\theta}\right\|^2, \qquad \forall\, t \geq T \qquad (5.42)\\ &\leq -\alpha\,||\zeta||^2 \qquad (5.43)\end{aligned}$$

where $\alpha = \min\Big(k_2\lambda_{min}(\bar{L}^2), k_3\gamma\Big)$ and $\zeta = [\tilde{s}^T\ \tilde{\theta}^T]^T$. Further, the Lyapunov function in (5.38) can be upper bounded as,

$$\begin{aligned}V &\leq \frac{\lambda_{max}(\bar{L})}{2}\,||\tilde{s}||^2 + \frac{\left\|\tilde{\theta}\right\|^2}{2} \\ &\leq max\Big(\frac{1}{2}, \frac{\lambda_{max}(\bar{L})}{2}\Big)\,||\zeta||^2, \qquad \forall t \geq T \qquad (5.44)\end{aligned}$$

Using (5.44) the inequality in (5.43) can be written as,

$$\dot{V} \leq -\eta V, \qquad \forall t \geq T \tag{5.45}$$

where $\eta = \frac{\alpha}{max\left(\frac{1}{2}, \frac{\lambda_{max}(\bar{L})}{2}\right)}$

$$V(t) \leq V(0)e^{-\eta t} \tag{5.46}$$

(5.46) implies that $\lim_{t\to\infty} V(t) \leq 0$. However from (5.38) we have $V(t) \geq 0$ implying $\lim_{t\to\infty} V(t) = 0 \implies \lim_{t\to\infty} ||\zeta|| = 0$. This in turn implies that $\lim_{t\to\infty} \tilde{s} = 0$ and $\lim_{t\to\infty} \tilde{\theta} = 0 \implies \lim_{t\to\infty} \tilde{\mathbf{b}} = 0$. Using these relations we have,

$$\dot{\mathbf{e}} + \lambda\mathbf{e} + \frac{\lambda\tilde{\mathbf{b}}}{2} = 0$$

$$\dot{\mathbf{e}} + \lambda\mathbf{e} = 0 \tag{5.47}$$

Solving (5.47) explicitly we get,

$$\mathbf{e}(t) = \mathbf{e}(0)e^{-\lambda t} \tag{5.48}$$

$$\lim_{t\to\infty} \mathbf{e}(t) = 0 \implies \lim_{t\to\infty} \dot{\mathbf{e}}(t) = 0 \tag{5.49}$$

□

Hence, using the proposed control law we are able to achieve exponential convergence of tracking error and bias estimation error to zero.

5.4 Choosing Control Gains

A sufficient condition for exponential convergence of bias estimation error to zero is the assumption that $Y_F(t)$ is exciting during a initial time window of finite length. A signal $y(t)$ being IE intuitively implies that it spans the entire m dimensional space as t varies from 0 to T. The IE condition is verified online by checking the determinant of $Y_{IF}(t)$; a positive value implying the fulfilment of the condition, thereafter, $Y_{IF}(t) \geq \gamma\, \forall t \geq T$ where $\gamma = \lambda_{min}(Y_{IF}(T))$. We have, $Y_F(t) = e^{-kt}\int_0^t e^{k\sigma}Y(\sigma)d\sigma$, where $Y(t) = \left[\ddot{\mathbf{q}} \quad M^{-1}k_1(\mathbf{q},\dot{\mathbf{q}},t)\bar{L}\dot{\mathbf{e}} \quad M^{-1}k_1(\mathbf{q},\dot{\mathbf{q}},t)\left(\bar{D} - \frac{\bar{L}}{2}\right)\right]$. Now, for regressor to satisfy IE condition Y_F must be full rank $\forall t \leq T$. To ensure this k_1 is made to be state and time dependent.

5.5 Simulation Results

In this section we present simulation results for a scenario where five two-link revolute joint arms (followers) track a leader with local interaction only using the proposed control law. The Laplacian and adjacency matrix along with leader connection of the agents are given below,

$$\mathcal{L} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{bmatrix}, \qquad \mathcal{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}, \qquad \bar{A} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \end{bmatrix}^T$$

The structure of the matrices are shown below,

$$\begin{aligned} M_i(\mathbf{q}_i) &= \begin{bmatrix} p_1 + 2p_3cos(\mathbf{q}_i^2) & p_2 + p_3cos(\mathbf{q}_i^2) \\ p_2 + p_3cos(\mathbf{q}_i^2) & p_2 \end{bmatrix} \\ C_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) &= \begin{bmatrix} -p_3sin(\mathbf{q}_i^2)\dot{\mathbf{q}}_i^2 & -p_3sin(\mathbf{q}_i^2)(\dot{\mathbf{q}}_i^1 + \dot{\mathbf{q}}_i^2) \\ p_3sin(\mathbf{q}_i^2)\dot{\mathbf{q}}_i^1 & 0 \end{bmatrix} \\ F_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) &= \begin{bmatrix} f_{d_1} & 0 \\ 0 & f_{d_2} \end{bmatrix} \end{aligned} \tag{5.50}$$

where $p_1 = 3.31$, $p_2 = .116$, $p_3 = .16$, $f_{d_1} = 5.3$, $f_{d_2} = 1.1$. The leader's trajectory is given by $\mathbf{q}_0 = [\sin(t)\,;\,\cos(t)]$. The initial position, velocity and the bias in relative measurement of the position for the i^{th} agent are given by $[\frac{i\pi}{7}\,;\,\frac{i\pi}{8}]$, $[0.1i-0.7\,;\,-0.1i+0.6]$ and $[\frac{i\pi}{12}\,;\,\frac{i\pi}{9}]$ respectively. $\hat{\theta}$ is initialized to zero vector. The gain constants k_2, k_3, λ, and k are chosen to be 1, 1.5, 0.4, 0.5 respectively and $k_1 == 4\text{diag}\Big(\sin(t\bar{L}\mathbf{e} + t\bar{D}\mathbf{b} + t\bar{L}\dot{\mathbf{e}})^2\Big) + 2\sin t^2$. Figures (5.1) to (5.3) show the convergence of the tracking error $\mathbf{e}_i(t)$, velocity error, $\dot{\mathbf{e}}_i(t)$, and bias estimation error $\tilde{\mathbf{b}}_i(t)$ to zero for all the followers.

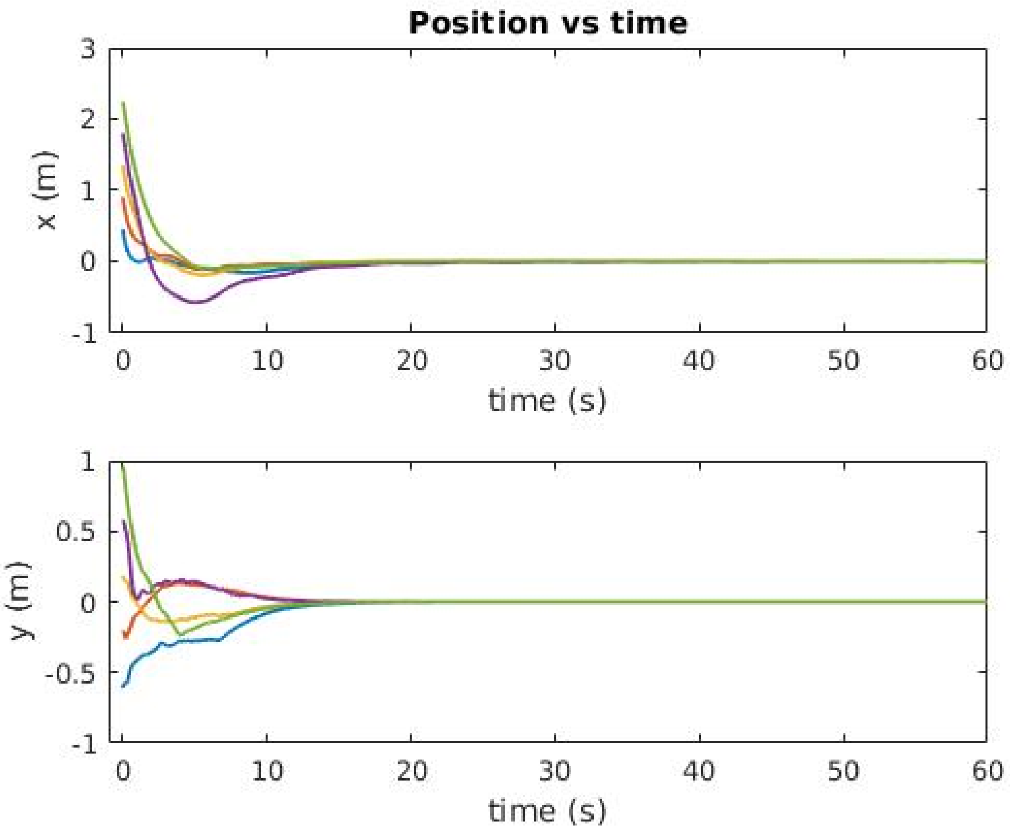

Figure 5.1: $\mathbf{e}(t)(m)$ vs time (s). The position of all agents converges exponentially to the leader's position

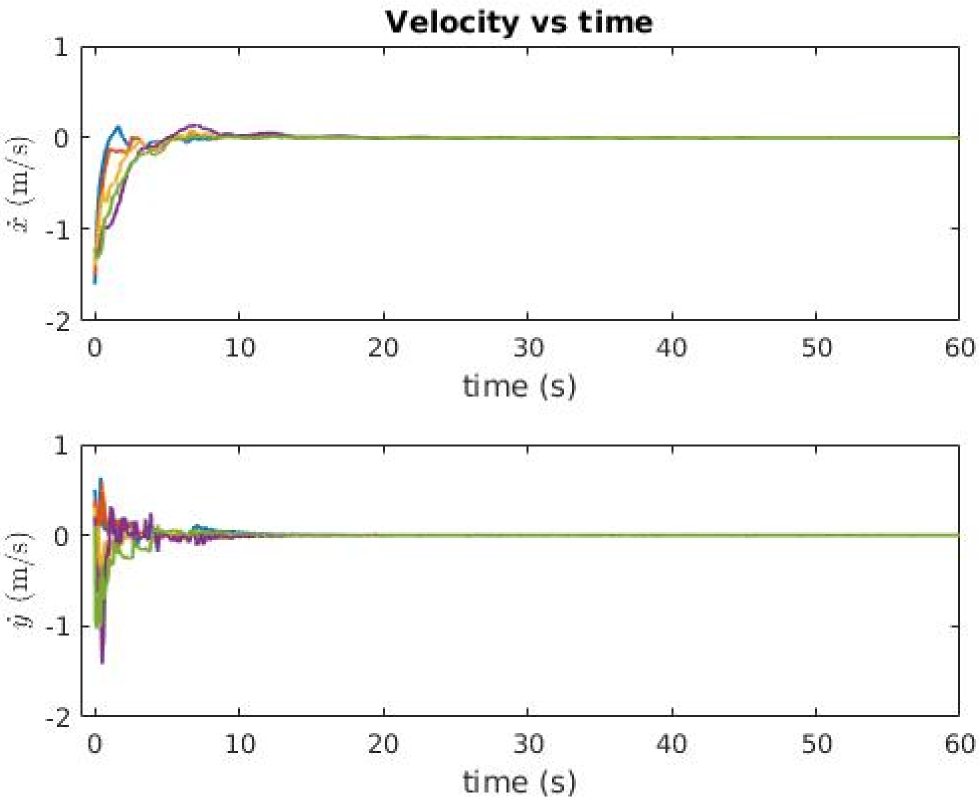

Figure 5.2: $\dot{\mathbf{e}}(t)(m/s)$ vs time (s). The velocity of all agents converges exponentially to the leader's velocity

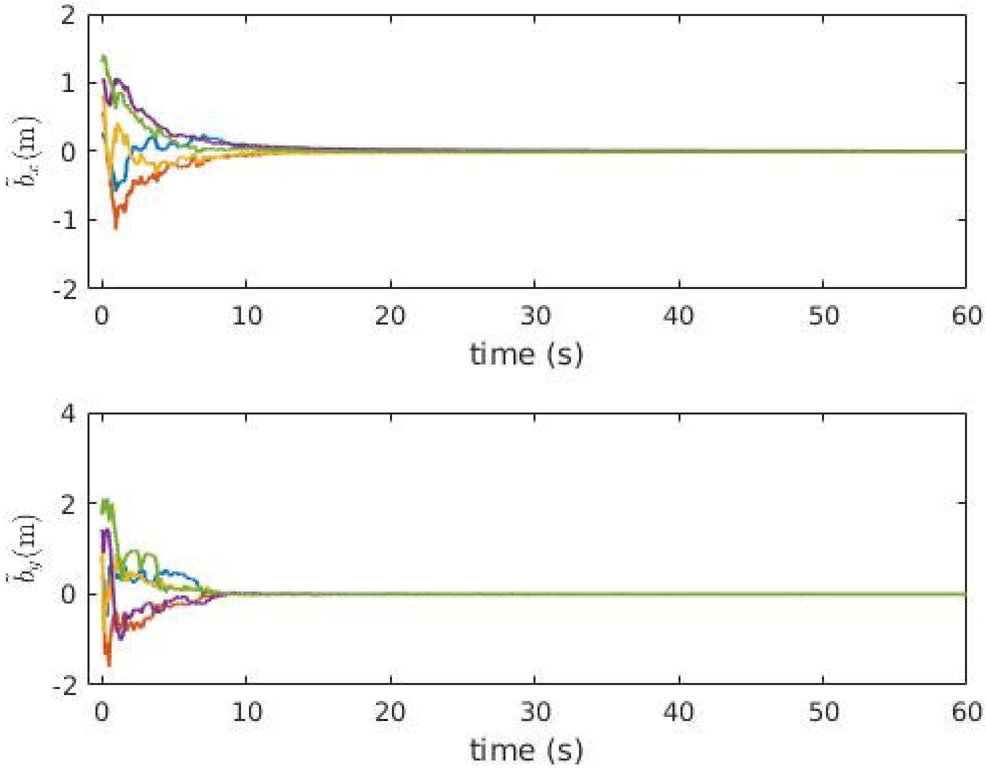

Figure 5.3: $\tilde{\mathbf{b}}(t)(m)$ vs time (s). The bias estimate error converges exponentially to zero for all agents

Chapter 6

Conclusion

A distributed algorithm is proposed in this project for an undirected connected network governed by Euler-Lagrange dynamics with biased measurements to achieve consensus tracking. No knowledge of upper bounds on the measurement errors are assumed in this work. Algorithms that lead to final algorithm are also discussed in detail. Using the first algorithm it is shown that the velocity and biased position with bias compensation exponentially converges to the leader's trajectory. This algorithm can be easily extended to any Euler-Lagrange system. Then, an algorithm based on a strategically designed composite adaptive controller is also tackled which significantly reduces the bound on consensus error. Finally, we develop a distributed continuous consensus control algorithm for an undirected connected certain networked Euler-Lagrange with biased relative position measurements. Control gains are designed to satisfy the IE condition on regressor. Simulations are then presented on a network of five two-link robot manipulator to show the effectiveness of the controller.

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List of Publications

Himani Sinhmar, Srikant Sukumar, *Distributed model independent algorithm for spacecraft synchronization under relative measurement bias*, 5^{th} CEAS Conference on Guidance, Navigation and Control, Italy

Himani Sinhmar, Srikant Sukumar, *Distributed consensus tracking of networked Euler-Lagrange systems under relative measurement bias*, under self revision for submission to Journal of Guidance, Control and Dynamics