

Consensus of networked double integrator systems under sensor bias

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Abstract—A novel distributed control law for consensus of networked double integrator systems with biased measurements is developed in this article. The agents measure relative positions over a time-varying, undirected graph with an unknown and constant sensor bias corrupting the measurements. An adaptive control law is derived using Lyapunov methods to estimate the individual sensor biases accurately. The proposed algorithm ensures that position consensus is achieved exponentially in addition to bias estimation. The results leverage recent advances in collective initial excitation based results in adaptive estimation. Conditions connecting bipartite graphs and collective initial excitation are also developed. The algorithms are illustrated via simulation studies on a network of double integrators with local communication and biased measurements.

Index Terms—Adaptive control, Multi-agent systems, Nonlinear control

I. INTRODUCTION

Consensus of networked double integrators has been studied extensively in control literature and several globally convergent controllers have been proposed [1], [2], [3], [4], [5]. This degree of interest is because the double integrator is one of the most fundamental block in any control system. Applications of double integrators include feedback linearizable nonlinear mechanical and aerospace systems such as free-rigid body motion, manipulator motion and spacecraft rotation. The consensus algorithms thus obtained can be further extended to complex nonlinear systems. Multi-agent systems with double integrator dynamics have been extensively studied in literature. See for example [6] and references therein. A comprehensive survey of the several consensus results in literature can be found in [7]. The preceding references, however, assume perfect measurements or extraneous disturbances only.

The motivation for the problem of consensus under sensor bias originates from mechanical systems that have only relative position and absolute velocity measurements available for feedback. However, relative position sensors suffer from errors such as bias in measurements. Unknown biases can appear during the functioning of various sensors such as rate gyros, accelerometers, magnetometers, altimeters, range sensors etc. These biases can be an outcome of inaccurate sensor calibration, environmental conditions, etc. The presence of bias deteriorates the performance of control laws on the network, and may result in stability issues [8], [9], [10]. Specifically, bias in relative position feedback could drive the agents to infinity, if not compensated. It is, therefore, of

interest to estimate the biases and possibly nullify their effect on the network. In the context of the continuous system, bias uncertainties in measurements are in general, sparsely studied. In the context of a single rigid-body system, ‘gyro bias’ is the most commonly addressed bias uncertainty and has been studied in detail in several references, including [11], [12], [13]. However, the literature on adaptive estimation and compensation of ‘position’ sensor bias is somewhat limited and the only relevant contributions known to the authors are by [14], [15]. There have of course been parallel approaches using non-smooth control laws, where disturbance rejection is possible for both single and networked second-order systems subject to knowledge of bounds on the bias uncertainty which is then modeled as a bounded disturbance. Such non smooth laws for disturbance rejection in double integrator systems have been explored in [16], [17], [18], [19]. An approach to estimating measurement inconsistencies using an output regulation-based technique is presented in [20]. Accurate estimation in [20], however requires a unique constant, graph structure.

For uncertain networked double integrators (sensor bias being one such uncertainty), conventional adaptive control laws (including [21], [22], [23], [24], [25]) require the regressor function to be persistently exciting (PE) or collectively persistently exciting (C-PE) for parameter convergence. Recently, several methods have been proposed to get rid of the PE condition for parameter convergence. [26] proposes an adaptive algorithm that uses both instantaneous state data and past measurements for the adaptation process. This scheme ensures parameter estimation errors converge to zero exponentially, subject to the satisfaction of a finite-time excitation condition. In the same spirit, [27] proposes a PI-like (Proportional-Integral controllers) parameter update law that guarantees parameter convergence with a relaxation of the PE condition, namely *initial excitation* (IE) on the regressor. [28] is an extension of [27], where the authors develop a distributed composite adaptive synchronization algorithm for multiple uncertain Euler-Lagrange (EL) systems to ensure parameter convergence using the *collective-IE* (C-IE) condition. The method proposed in [27] obviates the need for data-storage and memory allocation required in concurrent learning-based adaptive control [29] methods.

There have been several strides in consensus under bounded disturbance and zero-mean noise. In [30], a leader-follower consensus control for a network of double integrators is proposed for follower measurements corrupted by (zero-mean) noise. This control law ensures that consensus tracking is achieved in the mean square sense for both fixed and switching communication networks. However, no bias errors are accounted for in this work. [31] shows a Kalman filter inspired

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technique for consensus which is input to state stable. As would be expected, the accuracy of the cooperation objective is directly related to the power level of the communication noise. [32] analyzes the asymptotic properties of linear consensus algorithms in the presence of bounded measurement errors. Here, consensus is not guaranteed with respect to all possible noise realizations. In [33], a novel self-triggering co-ordination scheme for finite time consensus is proposed in the presence of unknown but bounded noise affecting the communication channels. Bias errors with unknown bounds are not the subject of study in any of the above articles.

We now summarize some of the results that lead up to the current work on consensus under measurement bias with unknown bounds. [34] proposes an adaptive control law in the presence of unknown constant bias for a double integrator network. This controller ensures bounded closed-loop signals in the presence of sensor bias which would not be the case in the absence of adaptation. However, convergence only to a neighborhood of consensus can be shown. [34] is based on the results in [14] which addresses the problem of accommodating unknown sensor bias in a direct MRAC setting for a single agent. In [35], the authors present a consensus algorithm for synchronization of double integrators over directed graphs in the presence of constant bias with unknown bounds. Here, the authors assume the existence of a bias error on each communication channel. Similar to [34], here too convergence ‘near’ a common equilibrium point is guaranteed. [36] shows an extension of [34], [35] to develop a distributed consensus tracking algorithm for spacecraft in formation modeled as an Euler-Lagrange network with similar bounded performance results. For a *fixed* communication graph, exact constant estimation of a constant bias and consensus in single integrator agents are demonstrated in [37]. An undirected, connected, and non-bipartite graph network is shown to be necessary and sufficient for estimation of the full bias vector.

In comparison with existing literature, the novel contributions of this article are as below.

- Adaptive control laws for exact bias estimation and consensus are developed over [34], [36], [35]. One sensor attached to each node is considered and *all* relative measurements from a node are assumed to be affected by the same bias in contrast to [35].
- A time-varying communication network topology is considered over [37], [34], [35], [36]. The analysis is based on jointly connected and jointly non-bipartite graphs.
- A non-bipartite property of the communication graph for a finite initial-time only is shown to be necessary in contrast to [37], where a constant non-bipartite graph is assumed for all time.
- A collective initial excitation based adaptive controller is employed for the first time in bias estimation problems over networks.

This paper is organized as follows. Section II introduces mathematical notation, necessary lemmas and in brief, graph theory. In section III, we formulate the consensus problem over a network of double integrator systems. We develop an adaptive control law for achieving consensus and bias

estimation in section IV. A discussion on the choice of control gains and the collective initial excitation condition on the regressor matrix are presented in section V. Section VI presents numerical simulations validating our algorithm. Conclusions are presented in section VII.

II. PRELIMINARIES

In this section, we present several mathematical notations, lemmas, assumptions, and a concise introduction of graph theory that forms the basis of the problem formulation.

A. Notation

\mathbb{R}^+ denotes non-negative reals. Kronecker product is denoted by \otimes . The Euclidean norm of a vector x is denoted by $\|x\|$ and the corresponding induced matrix Euclidean norm by $\|A\|$ for a matrix A . A diagonal matrix with elements d_1, d_2, \dots, d_n on the diagonal is represented by $\text{diag}(d_1, \dots, d_n)$. The $n \times n$ identity matrix and zero matrix are denoted by I_n and 0_n respectively; a n -dimensional vector of ones is denoted $\mathbf{1}_n$. For a matrix A , the maximum and minimum eigenvalues are respectively denoted by $\lambda_{max}(A)$ and $\lambda_{min}(A)$. For a symmetric matrix Γ , the notation $\Gamma > 0$ ($\Gamma < 0$) is used to denote a positive-definite (negative-definite) matrix. For a matrix signal, $A(\cdot) : \mathbb{R}^+ \rightarrow \mathbb{R}^{p_1 \times p_2}$, we define $\|A\|_\infty \triangleq \sup_{t \geq 0} \|A(t)\|$, the signal infinity norm. Time and initial condition arguments for all state variables (variables with dynamics) are uniformly omitted for notational simplicity. Similarly, function arguments for the control variables are suppressed, and they are made clear through explicit expressions proposed later in the manuscript.

B. Graph Theory

Consider a network of n agents interacting with each other over a time-varying graph. We define the interaction graph as a function of time ($t \geq 0$) through the tuple, $\mathcal{G}(t) \triangleq (\mathcal{V}, \mathcal{E}(t))$, where $\mathcal{V} \triangleq 1, \dots, n$ is a node set and $\mathcal{E}(t) \subseteq \mathcal{V} \times \mathcal{V}$ is an edge set signifying interaction between nodes [6] at time instant ‘ t ’. If an edge $(i, j) \in \mathcal{E}(t)$, then node i is called a *neighbor* of node j with j being the *head* node and i being the *tail* node indicating information flow from $i \rightarrow j$. The set of neighbors of a node i at time t , is denoted by $\mathcal{N}_i(t)$. In an undirected graph, $(j, i) \in \mathcal{E}(t) \Leftrightarrow (i, j) \in \mathcal{E}(t)$ for all $i, j \in \mathcal{V}$. An undirected graph $\mathcal{G}(t)$ is instantaneously *connected* if there is an undirected path between every pair of distinct nodes. The *adjacency* matrix, $\mathbb{R}^{n \times n} \ni \mathcal{A}(t) = [a_{ij}(t)]$, is defined such that $a_{ij}(t) > 0$ if $(j, i) \in \mathcal{E}(t)$ and $a_{ij}(t) = 0$ if $(j, i) \notin \mathcal{E}(t)$. We assume no self edges are present and hence, $a_{ii}(t) = 0$ for all t . For an undirected graph, \mathcal{A} is symmetric. The *degree* matrix of the graph \mathcal{G} is, $\mathcal{D}(t) \triangleq \text{diag}(\sum_{j=1}^n a_{1j}(t), \dots, \sum_{j=1}^n a_{nj}(t)) \in \mathbb{R}^{n \times n}$ and the *Laplacian* matrix, $\mathcal{L}(t) \triangleq [l_{ij}(t)] \in \mathbb{R}^{n \times n}$ is computed as:

$$\begin{aligned} \mathcal{L}(t) &= \mathcal{D}(t) - \mathcal{A}(t) \\ l_{ii}(t) &= \sum_{j=1, j \neq i}^n a_{ij}(t), \quad l_{ij}(t) = -a_{ij}(t), i \neq j \end{aligned}$$

As evident from above, $\mathcal{L}(t)$ is symmetric for undirected graphs. Further, $\mathcal{L}(t)$ has both row and column sums zero indicating that 0 is an eigenvalue with a corresponding eigenvector being $\mathbf{1}_n$ (vector of ones), i.e. $\mathcal{L}(t)\mathbf{1}_n = \mathbf{1}_n^\top \mathcal{L}(t) = 0$. Another symmetric matrix of interest is the *signless* Laplacian defined as $\mathcal{Q}(t) \triangleq \mathcal{D}(t) + \mathcal{A}(t) \triangleq [Q_{ij}(t)] \in \mathbb{R}^{n \times n}$ where,

$$Q_{ii}(t) = \sum_{j=1, j \neq i}^n a_{ij}(t), \quad Q_{ij}(t) = a_{ij}(t), i \neq j.$$

For an undirected graph $\mathcal{G}(t)$, both $\mathcal{L}(t)$ and $\mathcal{Q}(t)$ are positive semi-definite matrices. A *union graph* denoted $\cup_{\tau \in [t, t+T]} \mathcal{G}(\tau)$ is the graph formed by Adjacency matrix elements, $\bar{a}_{ij}(t) \triangleq \int_t^{t+T} a_{ij}(\tau) d\tau$. The union graph, as defined, is the graph obtained by collecting all the edges in the sub-graphs appearing over a time-interval $[t, t+T]$.

Definition 1. [38] For a time-varying graph $\mathcal{G}(t)$ with adjacency matrix elements $a_{ij}(t)$, the *weighted incidence matrix* $\mathcal{H} : \mathbb{R}^+ \rightarrow \mathbb{R}^{n \times \frac{n(n-1)}{2}}$ is defined as,

$$\mathcal{H}(t) \triangleq h_{ij}(t) \triangleq \begin{cases} \sqrt{a_{ij}(t)}, & \text{if } e_j = (i, j) \\ -\sqrt{a_{ij}(t)}, & \text{if } e_j = (j, i) \\ 0, & \text{otherwise} \end{cases}$$

Remark II.1. In the aforementioned definition, for undirected graphs, it is standard practice to choose an arbitrary orientation (information flow direction). This has no effect on the graph Laplacian and can always be computed as, $\mathcal{L}(t) = \mathcal{H}(t)\mathcal{H}^\top(t)$.

Definition 2. [39] At any given time ‘ t ’, an undirected graph $\mathcal{G}(t)$ is called *bipartite* if there exists a disjoint partition of the node set denoted as $\mathcal{V} = \mathcal{V}_+(t) \cup \mathcal{V}_-(t)$ such that all edges in $\mathcal{G}(t)$ are between the node sets, and there are no edges within the node set. Mathematically, for all $(i, j) \in \mathcal{E}(t)$, $i \in \mathcal{V}_k(t) \implies j \in \mathcal{V} \setminus \mathcal{V}_k(t)$ for $k \in \{+, -\}$. A graph is called *jointly (non-)bipartite* over $[t, t+T]$ if the corresponding union graph $\cup_{\tau \in [t, t+T]} \mathcal{G}(\tau)$ is (non-)bipartite.

Remark II.2. The above definition implies that the graph need not necessarily be (non-)bipartite for all time instants between $[t, t+T]$, but the graph obtained by collecting all the edges in the sub-graphs appearing over the time interval is (non-)bipartite.

Definition 3. [38] The time-varying graph $\mathcal{G}(t)$ is termed *jointly (δ, T) -connected* if there are two real numbers $\delta > 0$ and $T > 0$ such that the edges (j, i) satisfying,

$$\int_t^{t+T} a_{ij}(s) ds \geq \delta, \quad i, j \in \mathcal{V}$$

form a connected graph over \mathcal{V} for all $t \geq 0$.

Definition 4. (Persistence of Excitation) A locally integrable function $\phi : \mathbb{R}^+ \rightarrow \mathbb{R}^{n \times m}$ is said to be *persistently exciting* if there exist positive constants μ_1, μ_2 , and T such that,

$$\mu_1 I_n \leq \int_t^{t+T} \phi(\tau)\phi^\top(\tau) d\tau \leq \mu_2 I_n, \quad \forall t \geq 0$$

Definition 5. [27] (Initial Excitation) A locally integrable function $\phi : \mathbb{R}^+ \rightarrow \mathbb{R}^{p \times q}$ is said to be *initially exciting* if there exist constants $\bar{T}, \eta > 0$ such that,

$$\int_{t_0}^{t_0+\bar{T}} \phi^\top(\tau)\phi(\tau) d\tau \geq \eta I_q, \quad \text{some } t_0 \geq 0$$

The extension of persistence and initial excitation conditions to multi-agent systems are termed *collective persistence of excitation (C-PE)* and *collective initial excitation (C-IE)* respectively [28]. These are defined below.

Definition 6. A set of bounded, locally integrable signals $\phi_i : \mathbb{R}^+ \rightarrow \mathbb{R}^{r \times s}, \forall i = \{1, \dots, n\}$, are C-PE, if there exist constants $T > 0$ and $\gamma > 0$ such that,

$$\int_t^{t+T} \sum_{i=1}^n \phi_i(\tau)\phi_i(\tau)^\top d\tau \geq \gamma I_r, \quad \forall t \geq t_0 \geq 0$$

Definition 7. A set of bounded, locally integrable signals $\phi_i : \mathbb{R}^+ \rightarrow \mathbb{R}^{u \times v}, \forall i = \{1, \dots, n\}$, are C-IE, if there exist constants $\bar{T} > 0$ and $\gamma > 0$ such that,

$$\int_{t_0}^{t_0+\bar{T}} \sum_{i=1}^n \phi_i^\top(\tau)\phi_i(\tau) d\tau \geq \gamma I_v, \quad \text{some } t_0 \geq 0$$

The following assumption is intrinsic to the subsequent results.

Assumption 1. The network graph, $\mathcal{G}(t)$, is undirected and jointly (δ, T) -connected for some $\delta, T > 0$. We assume the same graph $\mathcal{G}(t)$ for both relative measurements as well as information exchange. Further, we assume existence of an $a_M > 0$ such that $a_{ij}(t) \leq a_M$ for all $i, j \in \{1, 2, \dots, n\}$ and for all $t \geq 0$.

C. Fundamental Results

We state a few results from graph theory and consensus analysis to be used subsequently.

Proposition II.1. [39] A graph \mathcal{G} is bipartite if and only if \mathcal{G} has no cycle of odd length.

Proposition II.2. [40] The smallest eigenvalue of the signless Laplacian matrix $\mathcal{Q} = \mathcal{D} + \mathcal{A}$ of an undirected and connected graph is equal to zero if and only if the graph is bipartite. In case the graph is bipartite, zero is a simple eigenvalue

The following is a re-wording of [41, Theorem 3.4].

Proposition II.3. Consider the time-varying dynamics,

$$\dot{x} = -\sigma N(t)N^\top(t)x, \quad x(0) = x_0 \quad (1)$$

with $N : \mathbb{R}^+ \rightarrow \mathbb{R}^{k \times p}$ being a piecewise continuous matrix function. If $N(\cdot)$ is persistently exciting (Definition 4), then the above dynamics admits a Lyapunov function

$$V(t, x) = \frac{1}{2} x^\top [\pi I_k + S(t)] x$$

where,

$$S(t) = 2\delta_T I_k - \frac{2}{T} \int_t^{t+T} \int_t^\tau N(\tau)N^\top(\tau) d\tau dr$$

and the positive constants π, δ_T are defined as,

$$\delta_T \triangleq T|N(\cdot)N^\top(\cdot)|_\infty,$$

$$\pi \triangleq 1 + \frac{2\sigma^2\delta_T^3}{\mu_1}.$$

Further, the states of the dynamical system (1) are uniformly exponentially stable at the origin.

The following result was established as part of the proof in [38, section IIA].

Proposition II.4. *The following hold for an undirected graph $\mathcal{G}(t)$ over n - nodes, with Laplacian $\mathcal{L}(t)$ and weighted incidence matrix $\mathcal{H}(t)$ (Definition 1).*

- $\left(\mathcal{L}(t) + \frac{\mathbf{1}_n \mathbf{1}_n^\top}{n}\right) = \left(\mathcal{H}(t) + \frac{\mathbf{1}_n h^\top(t)}{\sqrt{n}}\right) \left(\mathcal{H}(t) + \frac{\mathbf{1}_n h^\top(t)}{\sqrt{n}}\right)^\top$, where $h(t)$ is unit vector in the kernel of $\mathcal{H}(t)$.
- The graph $\mathcal{G}(t)$ is jointly (δ, T) -connected (Definition 3) for some $\delta, T > 0$ if and only if $\left(\mathcal{H}(t) + \frac{\mathbf{1}_n h^\top(t)}{\sqrt{n}}\right)$ is persistently exciting (Definition 4).

III. BIAS ESTIMATED CONSENSUS

The objective of this article is to develop a distributed consensus algorithm for a network of double integrator systems in the presence of a constant unknown bias corrupting the relative measurements of position while ensuring estimation of all biases by each agent. The interaction between the agents is modeled by an undirected and jointly (δ, T) -connected graph, $\mathcal{G}(t)$, with an associated Laplacian $\mathcal{L}(t)$. The input-output model for each agent representing a node in $\mathcal{G}(t)$ is expressed as the following double integrator equation,

$$\begin{aligned} \ddot{q}_i &= u_i, \\ q_i(0) &= q_{i0}; \dot{q}_i(0) = \dot{q}_{i0}; i = 1, 2, \dots, n \\ y_i &= [z_{ij}, \dot{q}_i]^\top, \quad i = 1, 2, \dots, n, j \in \mathcal{N}_i(t) \end{aligned} \quad (2)$$

where state $q_i \in \mathbb{R}^m$ is the vector of generalized coordinates (called ‘positions’ in general with their derivatives being ‘velocities’), $u_i(\cdot) \in \mathbb{R}^m$ is a distributed, time-varying feedback, $y_i = [z_{ij}, \dot{q}_i] \in \mathbb{R}^{2m}$ is the information state available to each agent with $\mathbb{R}^m \ni z_{ij} \triangleq (q_i - q_j + b_i)$ and $b_i \in \mathbb{R}^m$ is the constant, unknown sensor bias. The estimate of sensor bias (b_i) for each agent k will be denoted \hat{b}_i^k in the sequel. Each agent k has an estimate of all sensor biases ($\hat{b}^k = [\hat{b}_1^k, \hat{b}_2^k, \dots, \hat{b}_n^k]^\top$) and a dynamic update law is designed for the same.

The control objective in this article is to design a distributed feedback u_i so that the closed-loop solutions of (2) satisfy,

$$\begin{aligned} \lim_{t \rightarrow \infty} (q_i - q_j) &= 0, \quad \forall i, j \in \{1, 2, \dots, n\} \\ \lim_{t \rightarrow \infty} \dot{q}_i &= 0 \quad \forall i \in \{1, 2, \dots, n\} \\ \lim_{t \rightarrow \infty} (b_i - \hat{b}_i^k) &= 0 \quad \forall i, k \in \{1, 2, \dots, n\} \end{aligned} \quad (\text{Bias Estimated Consensus})$$

The following assumption delineates the information available for the design of the feedback law u_i .

Assumption 2. Agents can measure their own velocities (\dot{q}_i); neighbors measure relative velocities ($\dot{q}_i - \dot{q}_j$) and relative position corrupted by a constant unknown bias ($z_{ij} = q_i -$

$q_j + b_i$). Further, neighbors exchange their measurement of relative positions ($z_{ji} = q_j - q_i + b_j$) and their estimate of biases (\hat{b}^j) with each other.

IV. CONTROL LAW DESIGN

We prescribe a controller u_i for (2) to satisfy our control objective (Bias Estimated Consensus) where $b_i, i = 1, 2, \dots, n$ are assumed to be *unknown, constant* measurement biases. In this article, we consider the following distributed control algorithm,

$$u_i = k(t) \left(-\dot{q}_i - \frac{1}{2} \sum_{j \in \mathcal{N}_i} a_{ij}(t) [z_{ij} + z_{ji}] \right) + w_i$$

where, $k : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is a positive valued uniformly bounded function (there exists $k_M > 0$ such that $k(t) \leq k_M$ for all $t \geq 0$) to be prescribed later, $q = [q_1, \dots, q_n]^\top$ and $w_i(\cdot) \in \mathbb{R}^m$ is an auxiliary control term. The implementation of the term z_{ji} in the control law requires that all neighbors’ relative position measurements (corrupted by bias) be communicated to each agent. This is guaranteed by Assumption 2. The individual control expressions above can now be collected to specify the feedback for the entire double integrator network as follows:

$$u = -k(t)\dot{q} + \frac{k(t)}{2} [\mathcal{L}(t) \otimes I_m] b - k(t) [\mathcal{D}(t) \otimes I_m] b + w \quad (3)$$

where, u, b and w are the column stacked vectors of $[u_1, \dots, u_n]^\top, [b_1, \dots, b_n]^\top$ and $[w_1, \dots, w_n]^\top$ respectively. Let $\bar{\mathcal{L}}(t) \triangleq \mathcal{L}(t) \otimes I_m$ and $\bar{\mathcal{D}}(t) := \mathcal{D}(t) \otimes I_m$, then (3) can be simplified as,

$$u = -k(t)\bar{\mathcal{D}}(t)b - k(t)\dot{q} + \frac{k(t)}{2}\bar{\mathcal{L}}(t)b + w \quad (4)$$

We also obtain the network dynamics from (2) as,

$$\ddot{q} = u. \quad (5)$$

Substituting the control law (4) in (5), we obtain the following closed-loop network dynamics,

$$\ddot{q} + k(t)\dot{q} + k(t) \left(\bar{\mathcal{D}}(t) - \frac{1}{2}\bar{\mathcal{L}}(t) \right) b = w. \quad (6)$$

It is worth noting that $\bar{\mathcal{D}}(t) - \bar{\mathcal{L}}(t)/2 = \mathcal{Q}(t)/2 \otimes I_m \triangleq \bar{\mathcal{Q}}(t)/2$. (6) can be written in a standard regressor-parameter form as,

$$\underbrace{\begin{bmatrix} \ddot{q}, & k(t)\dot{q}, & \frac{1}{2}k(t)\bar{\mathcal{Q}}(t) \end{bmatrix}}_{Y \in \mathbb{R}^{mn \times (mn+2)}} \underbrace{\begin{bmatrix} 1 & 1 & b \end{bmatrix}^\top}_{\theta \in \mathbb{R}^{mn+2}} = w$$

with $Y : \mathbb{R}^m \times \mathbb{R}^m \times \mathbb{R} \rightarrow \mathbb{R}^{mn \times (mn+2)}$ denoting the regressor and $\theta \in \mathbb{R}^{mn+2}$ being the constant, unknown parameter. We can also write, corresponding to each agent:

$$Y_i = \left[\ddot{q}_i, k(t)\dot{q}_i, \frac{1}{2}k(t)[Q_{i1}(t)I_m, Q_{i2}(t)I_m, \dots, Q_{in}(t)I_m] \right]. \quad (7)$$

Y_i and w_i are available for each agent, i.e., $Y_i \in \mathbb{R}^{m \times (mn+2)}$ corresponds to each m rows of Y , $w_i \in \mathbb{R}^m$ corresponds to each m rows of w . Additionally, we have corresponding to each agent, $Y_i \theta = w_i$. Each agent has an adaptive estimate of the unknown parameter vector θ for all $i = 1, \dots, n$ (which

is an over-parametrization of b) denoted $\hat{\theta}^i \in \mathbb{R}^{mn+2}$. $\hat{\theta}^i = [p^i \ l^i \ \hat{b}^i]^\top$ where $\hat{b}^i = [\hat{b}_1^i, \hat{b}_2^i, \dots, \hat{b}_n^i]^\top$. p^i, l^i are the estimates of the constants in θ by agent i . We define the agent parameter error as, $\tilde{\theta}^i \triangleq [1 - p^i \ 1 - l^i \ \tilde{b}^i]^\top$, where $\tilde{b}^i \triangleq [b_1 - \hat{b}_1^i, \dots, b_n - \hat{b}_n^i]^\top = [\tilde{b}_1^i, \dots, \tilde{b}_n^i]^\top$. We now define,

$$s_i \triangleq \dot{q}_i + \lambda \left(q_i + \frac{\tilde{b}_i^i}{2} \right), \quad \lambda > 0, i = 1, 2, \dots, n \quad (8)$$

where $\tilde{b}_i^i \triangleq b_i - \hat{b}_i^i$. Further we assign, $s \triangleq [s_1, \dots, s_n]^\top$, $\tilde{b}_{new} \triangleq [\tilde{b}_1^1, \dots, \tilde{b}_n^n]^\top$ and obtain $s = \dot{q} + \lambda(q + \tilde{b}_{new}/2)$.

Note - \tilde{b}_i^i has been used in eq. (8), and not \tilde{b}_i^k as $\tilde{b}_i^k = b_i - \hat{b}_i^k$ will require bias estimate information of b_i computed by agent $k \notin \mathcal{N}_i$, which may not be available with agent i . Taking the derivative of s and substituting from (6),

$$\begin{aligned} \dot{s} &= \ddot{q} + \lambda \left(\dot{q} + \frac{\dot{\tilde{b}}_{new}}{2} \right) \\ &= -k(t)\dot{q} - \frac{1}{2}k(t)\bar{Q}(t)b + w + \lambda \left(\dot{q} + \frac{\dot{\tilde{b}}_{new}}{2} \right) \\ &= \underbrace{\begin{bmatrix} 0_{mn \times 1} & 0_{mn \times 1} & -\frac{1}{2}k(t)\bar{Q}(t) \end{bmatrix}}_{Z \in \mathbb{R}^{mn \times (mn+2)}} \begin{bmatrix} 1 & 1 & b \end{bmatrix}^\top \\ &\quad - k(t)\dot{q} + w + \lambda \left(\dot{q} + \frac{\dot{\tilde{b}}_{new}}{2} \right). \end{aligned} \quad (9)$$

The corresponding dynamics for each agent $s_i \in \mathbb{R}^m$, is given by

$$\dot{s}_i = Z_i \theta - k(t)\dot{q}_i + w_i + \lambda \left(\dot{q}_i + \frac{\dot{\tilde{b}}_i^i}{2} \right)$$

where $Z_i \in \mathbb{R}^{m \times (mn+2)}$ is defined similar to Y_i . We now define the second part of the control, w at each agent node as,

$$w_i = k(t)\dot{q}_i - Z_i \hat{\theta}^i - \lambda \left(\dot{q}_i + \frac{\dot{\tilde{b}}_i^i}{2} \right) - \sum_{j \in \mathcal{N}_i} a_{ij}(t)(s_i - s_j), \quad (10)$$

which can be written in an implementable form as,

$$\begin{aligned} w_i &= k(t)\dot{q}_i - \lambda \dot{q}_i - \frac{\lambda \dot{\tilde{b}}_i^i}{2} + k(t) \sum_{j \in \mathcal{N}_i} a_{ij}(t) \hat{b}_i^i \\ &\quad - \sigma \sum_{j \in \mathcal{N}_i} a_{ij}(t)(s_i - s_j) - \frac{k(t)}{2} \sum_{j \in \mathcal{N}_i} a_{ij}(t)(\hat{b}_i^i - \hat{b}_j^j), \quad \sigma > 0. \end{aligned} \quad (11)$$

The above yields for the entire network the following,

$$w = k(t)\dot{q} - \lambda(\dot{q} + \frac{\dot{\tilde{b}}_{new}}{2}) - \sigma \bar{L}(t)s - Z_{new} \hat{\theta} \quad (12)$$

where $\hat{\theta} := [\hat{\theta}^1, \dots, \hat{\theta}^n]^\top$ and $Z_{new}(\dot{q}, t) \in \mathbb{R}^{mn \times n(mn+2)}$ is defined as

$$Z_{new}(\dot{q}, t) \triangleq \text{diag}(Z_1(\dot{q}_1, t), Z_2(\dot{q}_2, t), \dots, Z_n(\dot{q}_n, t)).$$

Since the bias, b , is constant we have, $\dot{\tilde{b}}_i^i = -\dot{\hat{b}}_i^i$ and hence is implementable in (11). Further, though s_i is not implementable (due to the $(q_i + \tilde{b}_i^i/2)$ term in s_i), $(s_i - s_j) =$

$\dot{q}_i - \dot{q}_j + \frac{1}{2}\lambda(z_{ij} - z_{ji} - (\hat{b}_i^i - \hat{b}_j^j))$ is, and that is what appears in the control law (11). Further, the implementation of the term $(\hat{b}_i^i - \hat{b}_j^j)$ requires neighbors to exchange their bias estimates (Assumption 2).

The control law u_i after substituting for w_i from (11) is given by:-

$$\begin{aligned} u_i &= \frac{k(t)}{2} \sum_{j \in \mathcal{N}_i} a_{ij}(t)(\tilde{b}_i^i - \tilde{b}_j^j) - \lambda \dot{q}_i - \frac{\lambda \dot{\tilde{b}}_i^i}{2} \\ &\quad - \sigma \sum_{j \in \mathcal{N}_i} a_{ij}(t)(s_i - s_j) - k(t) \sum_{j \in \mathcal{N}_i} a_{ij}(t)(\tilde{b}_i^i) \end{aligned}$$

Substituting (10) in (9),

$$\dot{s} = Z_{new} \tilde{\theta} - \sigma \bar{L}(t)s \quad (13)$$

where $\tilde{\theta} := [\tilde{\theta}^1, \dots, \tilde{\theta}^n]^\top$, $\tilde{\theta}^i = \theta - \hat{\theta}^i$.

A. Bias Estimation

The matrix $Y_i(\dot{q}_i, \ddot{q}_i, t)$ in (7) is dependent on the acceleration term \ddot{q}_i and so cannot be used in our adaptation law for bias estimation. In order to facilitate relaxation of the persistence of excitation condition, a filter is designed for each agent as proposed in [28],

$$\begin{aligned} \dot{Y}_{F_i} &= -\beta Y_{F_i} + Y_i(\dot{q}_i, \ddot{q}_i, t), & Y_{F_i}(0) &= 0 \\ \dot{w}_{F_i} &= -\beta w_{F_i} + w_i, & w_{F_i}(0) &= 0 \end{aligned} \quad (14)$$

where $\beta > 0$ is the scalar filter gain, $Y_{F_i} \in \mathbb{R}^{m \times (mn+2)}$ and $w_{F_i} \in \mathbb{R}^m$. Solving the above equations explicitly we obtain,

$$Y_{F_i}(t) = e^{-\beta t} \int_0^t e^{\beta r} Y_i(\dot{q}_i, \ddot{q}_i, r) dr \quad (15)$$

$$w_{F_i}(t) = e^{-\beta t} \int_0^t e^{\beta r} w_i dr. \quad (16)$$

Utilizing the relation $Y_i(\dot{q}_i, \ddot{q}_i, t)\theta = w_i$ we get $Y_{F_i}\theta = w_{F_i}$ from (15) and (16). Y_{F_i} in (14) cannot be solved explicitly as $Y_i(\dot{q}_i, \ddot{q}_i, t)$ is not measured. Therefore, we split $Y_i(\dot{q}_i, \ddot{q}_i, t)$ into measured and non-measured parts as,

$$Y_i(\dot{q}_i, \ddot{q}_i, t) = Y_{1_i}(\ddot{q}_i) + Y_{2_i}(\dot{q}_i, t)$$

where

$$Y_{1_i}(\ddot{q}_i) = \begin{bmatrix} \ddot{q}_i & 0_{m \times 1} & 0_{m \times mn} \end{bmatrix}$$

$$Y_{2_i}(\dot{q}_i, t) =$$

$$\begin{bmatrix} 0_{m \times 1} & k(t)\dot{q}_i & \frac{1}{2}k(t)[Q_{i1}(t)I_m, Q_{i2}(t)I_m, \dots, Q_{in}(t)I_m] \end{bmatrix}$$

This implies that $Y_{F_i} = Y_{F_{1_i}} + Y_{F_{2_i}}$, where,

$$\begin{aligned} \dot{Y}_{F_{1_i}} &= -\beta Y_{F_{1_i}} + Y_{1_i}, & Y_{F_{1_i}}(0) &= 0 \\ \dot{Y}_{F_{2_i}} &= -\beta Y_{F_{2_i}} + Y_{2_i}, & Y_{F_{2_i}}(0) &= 0 \end{aligned} \quad (17)$$

Since $Y_{2_i}(\dot{q}_i, t)$ is known, $Y_{F_{2_i}}$ can be solved online using (17) by employing a numerical integration scheme. We solve $Y_{F_{1_i}}$ analytically as follows,

$$\begin{aligned} Y_{F_{1_i}}(t) &= e^{-\beta t} \int_0^t e^{\beta r} Y_{1_i}(\ddot{q}(r)) dr \\ &= \begin{bmatrix} e^{-\beta t} \int_0^t e^{\beta r} \ddot{q}_i(r) dr & 0 & 0 \end{bmatrix} \end{aligned}$$

The elements of $Y_{F_{1_i}}$ can be evaluated using integration by parts as follows,

$$Y_{F_{1_i}}(t) = \begin{bmatrix} e^{-\beta t} [e^{\beta t} \dot{q}_i(t) - \dot{q}_i(0)] - e^{-\beta t} \int_0^t \beta e^{\beta r} \dot{q}_i(r) dr & 0 \\ \dot{q}_i(t) - e^{-\beta t} \dot{q}_i(0) - h_i(t) & 0 \end{bmatrix}$$

$\forall i = 1, \dots, n$ and

$$\dot{h}_i = \beta \dot{q}_i - \beta h_i, \quad h_i(0) = 0$$

Y_{F_i} , w_{F_i} are filtered regressor and filtered control for each agent i respectively. Y_{F_i} can now be used in our adaptation law. Additionally, for bias estimation, we will make use of the double filtered regressor and control law introduced in [27], [28]

$$\dot{Y}_{IF_i} = Y_{F_i}^\top Y_{F_i}, \quad Y_{IF_i}(0) = 0 \quad (18)$$

$$\dot{w}_{IF_i} = Y_{F_i}^\top w_{F_i}, \quad w_{IF_i}(0) = 0 \quad (19)$$

where $Y_{IF_i} \in \mathbb{R}^{(mn+2) \times (mn+2)}$, $w_{IF_i} \in \mathbb{R}^{(mn+2)}$.

Fact IV.1. [27] Integrating (18) and (19) and using the relation $Y_{F_i} \theta = w_{F_i}$ it can be verified that,

$$Y_{IF_i} \theta = w_{IF_i}, \quad \forall t \geq 0 \quad (20)$$

Fact IV.2. [27] The solution $Y_{IF_i}(t)$ of (18) is a non-negative and non-decreasing function of time.

The adaptive control law for bias estimation is now chosen as,

$$\begin{aligned} \dot{\hat{\theta}}^i &= \mu_F Y_{F_i}^\top (w_{F_i} - Y_{F_i} \hat{\theta}^i) + \mu_{IF} (w_{IF_i} - Y_{IF_i} \hat{\theta}^i) \\ &+ \sum_{j \in \mathcal{N}_i} a_{ij} (\hat{\theta}^i - \hat{\theta}^j), \quad \forall i = 1, \dots, n \end{aligned} \quad (21)$$

which using Fact IV.1 can be written as,

$$\begin{aligned} \dot{\hat{\theta}} &= -\mu_F \phi_F \tilde{\theta} - \mu_{IF} \phi_{IF} \tilde{\theta} - \mathcal{L} \otimes I_{(mn+2)} \tilde{\theta} \\ \dot{\hat{b}}^i &= [\hat{\theta}_{(3)}^i \quad \hat{\theta}_{(4)}^i \quad \dots \quad \hat{\theta}_{(mn+2)}^i]^\top \end{aligned} \quad (22)$$

for constant μ_F , $\mu_{IF} > 0$ and arbitrary initial conditions. $\hat{\theta}_{(k)}^i$ denotes the k^{th} -element of $\hat{\theta}^i$ and so on. $\phi_F, \phi_{IF} \in \mathbb{R}^{n(mn+2) \times n(mn+2)}$ are block diagonal matrices and are defined as,

$$\begin{aligned} \phi_F &\triangleq \text{diag}(Y_{F_1}^\top Y_{F_1}, \dots, Y_{F_n}^\top Y_{F_n}) \\ \phi_{IF} &\triangleq \text{diag}\left(\int_0^t Y_{F_1}^\top Y_{F_1}, \dots, \int_0^t Y_{F_n}^\top Y_{F_n}\right). \end{aligned}$$

Further, we consider the following assumption and a corresponding proposition.

Assumption 3. The set of filtered regressors Y_{F_i} are C-IE as per Definition 7.

Remark IV.1. It is worth noting that the solution Y_{F_i} in the assumption above depends on the initial conditions of the closed-loop state $\dot{q}_i(0)$. The collective initial excitation condition is therefore not necessarily uniform with respect to initial data.

Proposition IV.1. [28] *Provided Assumption 3 holds, the matrix $M(t) = \mathcal{L} \otimes I_{(mn+2)} + \mu_{IF} \phi_{IF}$ appearing in (22)*

is uniformly strictly positive definite over the time window $[\bar{T}, \infty)$ i.e.,

$$\begin{aligned} \xi^\top M(t) \xi &> 0, \quad \forall t \geq \bar{T} \\ \forall \xi &\in \mathbb{R}^{n(mn+2)}. \end{aligned}$$

We are now ready to state the primary result of this article.

Theorem IV.1. *Consider the multi-agent network with the agent dynamics given by (2) interacting over an undirected graph $\mathcal{G}(t)$. If Assumptions 1-3 hold, then the control law given by,*

$$u = Z_{new} \tilde{\theta} - \sigma \bar{\mathcal{L}}(t) s - \lambda \dot{q} - \lambda \frac{\dot{b}_{new}}{2}$$

with bias adaptation law (21), guarantees that $\lim_{t \rightarrow \infty} (q_i - q_j) = 0$ (for all $i, j \in \{1, 2, \dots, n\}$), $\lim_{t \rightarrow \infty} \dot{q} = 0$ and $\lim_{t \rightarrow \infty} (b_i - \hat{b}_i^k) = 0$ (for all $i, k \in \{1, 2, \dots, n\}$) exponentially (beyond the collective initial excitation window, i.e. $t \geq \bar{T}$) for sufficiently large $\mu_{IF} > 0$, while ensuring that the trajectories of the closed-loop system given by (9), (12), and (22) are uniformly bounded.

Remark IV.2. While the result stated in Theorem IV.1 pertains to the consensus problem, the same idea extends to the trajectory tracking problems for a known bounded, smooth trajectory $r(t)$ known to the agents. The error variable $e \triangleq (q - r)$ is used in the results and control design instead of q .

Proof. The closed-loop system using (13), (22), and (20) can be written in the following matrix structure,

$$\begin{aligned} \frac{d}{dt} \begin{pmatrix} s \\ \hat{\theta} \end{pmatrix} &= \begin{pmatrix} -\sigma I_{mn} & Z_{new}(\dot{q}, t) \\ 0 & -\mu_F \phi_F - \mu_{IF} \phi_{IF} - \mathcal{L} \otimes I_{(mn+2)} \end{pmatrix} \\ &\times \begin{pmatrix} \bar{\mathcal{L}}(t) s \\ \hat{\theta} \end{pmatrix} \end{aligned}$$

We now define a new consensus error variable $\epsilon \triangleq (I_{mn} - \frac{\mathbf{1}_n \mathbf{1}_n^\top}{n} \otimes I_m) s = (s - \sum_{i=1}^n s_i / n)$. The dynamics in the new error state variables are,

$$\begin{aligned} \dot{\epsilon} &= -\sigma \left(\bar{\mathcal{L}}(t) + \frac{\mathbf{1}_n \mathbf{1}_n^\top}{n} \otimes I_m \right) \epsilon \\ &+ \left(I_{mn} - \frac{\mathbf{1}_n \mathbf{1}_n^\top}{n} \otimes I_m \right) Z_{new}(\dot{q}, t) \tilde{\theta} \\ \dot{\hat{\theta}} &= -(\mu_F \phi_F + \mu_{IF} \phi_{IF} + \mathcal{L} \otimes I_{(mn+2)}) \tilde{\theta} \end{aligned} \quad (23)$$

Arriving at the first equation is a straightforward application of the definition of ϵ , computing its derivative according to the preceding dynamics and noting the fact that $\sigma \bar{\mathcal{L}}(t) s = \sigma \bar{\mathcal{L}}(t) (\epsilon + \frac{\mathbf{1}_n \mathbf{1}_n^\top}{n} \otimes I_m s)$; $(\frac{\mathbf{1}_n \mathbf{1}_n^\top}{n} \otimes I_m) (I_{mn} - \frac{\mathbf{1}_n \mathbf{1}_n^\top}{n} \otimes I_m) = 0$. Similar transformation equations appear in consensus analysis in [38, section IIA].

By applying Proposition II.4 we have,

$$\left(\bar{\mathcal{L}}(t) + \frac{\mathbf{1}_n \mathbf{1}_n^\top}{n} \otimes I_m \right) = (N(t) \otimes I_m) (N(t) \otimes I_m)^\top$$

where $N(t) \triangleq \left(\mathcal{H}(t) + \frac{\mathbf{1}_n \mathbf{h}^\top(t)}{\sqrt{n}} \right)$. It is also evident from Proposition II.4 that $N(t)$ is persistently exciting since $\mathcal{G}(t)$ is jointly (δ, T) -connected. For the error dynamics (23) we

choose the following candidate Lyapunov function motivated by Proposition II.3,

$$V(t, \epsilon, \tilde{\theta}) = \frac{1}{2} \epsilon^\top [\pi I_{mn} + S(t)] \epsilon + \frac{1}{2} \tilde{\theta}^\top \tilde{\theta}$$

where $S(t)$ and constants π, δ_T are as defined in Proposition II.3 with $N(t)$ defined above. It is immediately evident from the definition of $S(t)$ and δ_T that,

$$0 \leq S(t) \leq 2\delta_T I_{mn}$$

and the derivative of $S(t)$ can be computed as,

$$\dot{S}(t) = 2N(t)N^\top(t) - \frac{2}{T} \int_t^{t+T} N(\tau)N^\top(\tau) d\tau.$$

It is therefore obvious that $V(t, \epsilon, \tilde{\theta}) \geq 0.5(\pi \|\epsilon\|^2 + \|\tilde{\theta}\|^2)$ and therefore is a positive definite function. The directional derivative of $V(t, \epsilon, \tilde{\theta})$ along the dynamics (23) can now be computed as,

$$\begin{aligned} \dot{V}(t, \epsilon, \tilde{\theta}) &= -(\pi\sigma - 1)\epsilon^\top N(t)N^\top(t)\epsilon - \sigma\epsilon^\top S(t)N(t)N^\top(t)\epsilon \\ &\quad - \frac{1}{T}\epsilon^\top \int_t^{t+T} N(\tau)N^\top(\tau) d\tau\epsilon \\ &\quad + \epsilon^\top [\pi I_{mn} + S(t)] \bar{I}_{mn} Z_{new}(\dot{q}, t) \tilde{\theta} \\ &\quad - \tilde{\theta}^\top (\mu_F \phi_F + \mu_{IF} \phi_{IF} + \mathcal{L} \otimes I_{(mn+2)}) \tilde{\theta} \end{aligned}$$

where $\bar{I}_{mn} \triangleq \left(I_{mn} - \frac{1_n 1_n^\top}{n} \otimes I_m \right)$ is used as a placeholder. We now compute an upper bound for $\dot{V}(t, \epsilon, \tilde{\theta})$ as below. Keeping in mind that $\phi_F \geq 0$, $\phi_{IF} \geq 0$ and Proposition IV.1 we obtain,

$$\begin{aligned} \dot{V}(t, \epsilon, \tilde{\theta}) &\leq -(\pi\sigma - 1)\|N^\top(t)\epsilon\|^2 - \frac{\mu_1}{T}\|\epsilon\|^2 + \frac{\sigma\gamma}{2}\|N^\top(t)\epsilon\|^2 \\ &\quad + \frac{\sigma}{2\gamma}\|S(t)N(t)\|^2\|\epsilon\|^2 - \mu_{IF}\lambda_{min}(M(t))\|\tilde{\theta}\|^2 \\ &\quad + \|[\pi I_{mn} + S(t)]\|\bar{I}_{mn}\|Z_{new}(\dot{q}, t)\|\epsilon\|\|\tilde{\theta}\| \end{aligned}$$

$$\begin{aligned} &\leq -(\pi\sigma - 1 - \frac{\sigma\gamma}{2})\|N^\top(t)\epsilon\|^2 - \left(\frac{\mu_1}{T} - \frac{\sigma}{2\gamma}\|N\|_\infty^2\|S\|_\infty^2\right)\|\epsilon\|^2 \\ &\quad + z_M(\pi + \|S\|_\infty)\|\bar{I}_{mn}\|\|\epsilon\|\|\tilde{\theta}\| - \mu_{IF}\lambda_{min}(M(t))\|\tilde{\theta}\|^2 \end{aligned}$$

where the first inequality is based on norm upper bounding, utilizing the persistence condition on $N(t)$ (Definition 4) and applying the Young's inequality to bound mixed terms in ϵ using a constant $\gamma > 0$. We now note that $\|S\|_\infty \leq 2\delta_T$ and make the following choice of constants in the above inequality,

$$\gamma = \frac{4\sigma T \delta_T^2}{\mu_1} \|N\|_\infty^2; \quad \pi = \frac{1}{\sigma} + \frac{2\sigma T \delta_T^2}{\mu_1} \|N\|_\infty^2$$

which leads to,

$$\begin{aligned} \dot{V}(t, \epsilon, \tilde{\theta}) &\leq -\frac{\mu_1}{2T}\|\epsilon\|^2 + z_M(\pi + 2\delta_T)\|\bar{I}_{mn}\|\|\epsilon\|\|\tilde{\theta}\| \\ &\quad - \mu_{IF}\lambda_{min}(M(t))\|\tilde{\theta}\|^2 \\ &\leq -\left(\frac{\mu_1}{2T} - \frac{\beta}{2\gamma^\circ}\right)\|\epsilon\|^2 \\ &\quad - \left(\mu_{IF}\lambda_{min}(M(t)) - \frac{\beta\gamma^\circ}{2}\right)\|\tilde{\theta}\|^2 \end{aligned}$$

where the final inequality is an application of the Young's inequality with some $\gamma^\circ > 0$ and $\beta \triangleq z_M(\pi + 2\delta_T)\|\bar{I}_{mn}\|$. We

note that $M(t) \geq M(t_0 + \bar{T}) > 0 \forall t \geq t_0 + \bar{T}$, which implies $\exists c > 0$ such that $\lambda_{min}(M(t)) \geq c > 0$ using the argument as in Fact IV.2. Therefore, the following choice of constants guarantees exponential convergence of the $(\epsilon, \tilde{\theta})$ dynamics to the origin.

$$\gamma^\circ > \frac{\beta T}{\mu_1}; \quad \mu_{IF} > \frac{\beta^2 T}{2\mu_1 \lambda_{min}(M(t))}$$

We can now argue from the convergence of ϵ that $\bar{\mathcal{L}}(t)s \rightarrow 0$ exponentially. Now employing the definition of s_i in (8) and accounting for the fact that $s_i - s_j \rightarrow 0$ for all $i, j \in \{1, 2, \dots, n\}$ and $\tilde{b}^i \rightarrow 0$ exponentially, we are left with,

$$\frac{d}{dt}(q_i - q_j) = -\lambda(q_i - q_j) + \Upsilon(t)$$

where $\Upsilon : \mathbb{R}^+ \rightarrow \mathbb{R}^m$ denotes an exponentially decaying function. This immediately shows that $\lim_{t \rightarrow \infty} (q_i - q_j) = 0$ and the convergence is exponential. This implies that $(\bar{\mathcal{L}}(t)q, \bar{\mathcal{L}}(t)\dot{q}) \rightarrow (0, 0)$ exponentially, from the above equation. We use these facts to carry out an asymptotic analysis of the closed-loop by substituting w from (12) in (6). Since, $\tilde{b}, \dot{\tilde{b}} \rightarrow 0$, we have the dynamics, in the limit, as $\ddot{q} = -\lambda\dot{q}$, which immediately proves that $\lim_{t \rightarrow \infty} \dot{q} = 0$ exponentially using similar arguments as before.

For proof of boundedness, we note that in the $\tilde{\theta}$ dynamics of (23), $\mu_F, \mu_{IF} > 0$ and $\phi_F, \phi_{IF}, \mathcal{L} \otimes I_{(mn+2)}$ are symmetric positive semidefinite matrices at each $t \geq 0$. Therefore we immediately have $\|\tilde{\theta}\| \leq \|\tilde{\theta}(0)\|$. It is already known that the unforced ($\theta = 0$) dynamics of ϵ is exponentially stable ([41], [38]) and from the fact that the forcing term is bounded, we can conclude boundedness of $\|\epsilon\|$ irrespective of the collective initial excitation on Y_{F_i} . Therefore $\|\bar{\mathcal{L}}(t)s\| = \|\bar{\mathcal{L}}(t)\epsilon\|$ is also uniformly bounded. Therefore, employing the definition of s_i in (8) we obtain,

$$s_i - s_j = \dot{q}_i - \dot{q}_j + \lambda(q_i - q_j) = \psi(t)$$

where $\psi : \mathbb{R}^+ \rightarrow \mathbb{R}^{mn}$ is a uniformly bounded function ($\|\psi(t)\| \leq \psi_M$). We have used the boundedness of \tilde{b} to arrive at the above. Solving the above equation allows us to conclude that $\|(q_i - q_j)\|$ and $\|(\dot{q}_i - \dot{q}_j)\|$ are uniformly bounded. \square

Remark IV.3. We note that the first equation in the system (23) is identical to the consensus dynamics studied in [41] and similar in structure to [38, section IIA] if the forcing term due to $\tilde{\theta}$ vanishes. The $\tilde{\theta}$ term is a result of the unknown sensor bias being studied in this article and evolves according to network properties embedded in ϕ_F, ϕ_{IF} , and $Z_{new}(\dot{q}, t)$.

V. CHOOSING GAINS AND C-IE CONDITION ON REGRESSORS

A central condition for exponential convergence of the bias estimation error to zero is Assumption 3. We have, $Y_{F_i}(t) = e^{-\beta t} \int_0^t e^{\beta s} Y_i(\dot{q}_i(\tau), \ddot{q}_i(\tau), \tau) d\tau$, where from (7), $Y_i(\dot{q}_i, \ddot{q}_i, t) = \left[\ddot{q}_i, k(t)\dot{q}_i, \frac{1}{2}k(t)[Q_{i1}(t)I_m, Q_{i2}(t)I_m, \dots, Q_{in}(t)I_m] \right]$. For Assumption 3 to be satisfied, it is required that the integral of $\sum_{i=1}^n Y_{F_i}^\top Y_{F_i}$ over the initial finite time window spans the $mn + 2$ dimensional space. We will now prove that, if the set of regressors Y_i 's are C-IE then the set of filtered regressors,

Y_{F_i} 's, are also C-IE which further implies that Y_{IF_i} 's are C-IE.

Proposition V.1. *The sufficient condition for the set of Y_{F_i} 's ($i \in \{1, 2, \dots, n\}$) to be C-IE is that the set of Y_i 's are C-IE.*

Proof. We proceed along the line of proof given in [42, Proposition 4.1]. Consider an arbitrary unit vector $v \in \mathbb{R}^{(mn+2)}$ and define the following variables:-

$$\begin{aligned} K_i &\triangleq Y_i v \\ K_{F_i} &\triangleq Y_{F_i} v \end{aligned}$$

Let us assume that the set of regressors Y_i 's are C-IE. The above proposition can now be proved by contradiction. Suppose that Y_{F_i} 's are not C-IE. Then, $\exists v \in \mathbb{R}^{(mn+2)}$ such that

$$\int_{t_0}^{t_0+T} \sum_{i=1}^n (K_{F_i}^\top(\tau) K_{F_i}(\tau)) d\tau = 0$$

which implies that, $K_{F_i}(t) = 0, \forall i, \forall t \in [t_0, t_0 + T]$. Therefore, $\dot{K}_{F_i}(t) = 0 \forall i$ and $t \in (t_0, t_0 + T)$. By definition, we have,

$$\dot{K}_{F_i} = -\beta K_{F_i} + K_i$$

which indicates that $K_i(t)$ is zero for all $i, \forall t \in (t_0, t_0 + T)$. This contradicts the fact that Y_i 's are C-IE. Hence, the set of Y_i 's being C-IE implies that the set of Y_{F_i} 's are C-IE. \square

We now derive a necessary condition to be able to conclude collective Initial Excitation (C-IE) on the set of regressors, $Y_i(\dot{q}, \ddot{q}, t)$. Since, $Y = [Y_1, Y_2, \dots, Y_n]^\top$ we can write,

$$\sum_{i=1}^n Y_i^\top Y_i(\dot{q}_i, \ddot{q}_i, t) = Y^\top Y(\dot{q}, \ddot{q}, t) = \begin{pmatrix} A(\dot{q}, \ddot{q}, t) & B(\dot{q}, \ddot{q}, t) \\ B^\top(\dot{q}, \ddot{q}, t) & C(\dot{q}, \ddot{q}, t) \end{pmatrix}$$

$$\begin{aligned} \text{where, } A(\dot{q}, \ddot{q}, t) &= \begin{pmatrix} \ddot{q}^\top \ddot{q} & k(t) \ddot{q}^\top \dot{q} \\ k(t) \dot{q}^\top \ddot{q} & k^2(t) \|\dot{q}\|^2 \end{pmatrix}, \\ B(\dot{q}, \ddot{q}, t) &= \begin{pmatrix} \frac{k(t)}{2} \ddot{q}^\top \bar{Q}(t) \\ \frac{k^2(t)}{2} \dot{q}^\top \bar{Q}(t) \end{pmatrix}, \\ C(t) &= \frac{k^2(t)}{4} \bar{Q}(t)^\top \bar{Q}(t). \end{aligned}$$

All arguments in the preceding equation have been deliberately removed for the sake of brevity and clarity. The functions $A(\dot{q}, \ddot{q}, t)$ and $C(t)$ as defined above, map into positive semidefinite matrices by definition. We now state the result pertinent to this section.

Lemma V.1. *If $Y_i(\dot{q}_i, \ddot{q}_i, t)$'s are collectively initially exciting as per Definition 7, then the matrix functions, $\int_0^T A(\dot{q}, \ddot{q}, t) dt$ and $\int_0^T C(t) dt$ are positive definite. Further, if the graph, $\mathcal{G}(t)$ is jointly (δ, T) -connected for some $\delta > 0$, then it is also jointly non-bipartite over $[0, \max\{T, \bar{T}\}]$.*

Proof. Let us assume for contradiction that $0 \in \text{spec}\{\int_0^T C(t) dt\}$. Let the eigenvalues be ordered as $0 \leq \beta_2 \leq \dots \leq \beta_{mn}$. We already know that $\int_0^T \sum_{i=1}^n Y_i^\top Y_i(\dot{q}_i, \ddot{q}_i, \tau) d\tau \geq 0$ which indicates that all eigenvalues are non-negative. Let us assume these are ordered as $\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_{(mn+2)}$. Since $\int_0^T C(t) dt$

is a principal submatrix of $\int_0^T \sum_{i=1}^n Y_i^\top Y_i(\dot{q}_i, \ddot{q}_i, \tau) d\tau$, we can use the Cauchy's interlacing and inclusion theorem ([43, Theorem 8.4.5]) to conclude that, $\lambda_1 \leq 0 \leq \lambda_3$. Since $\int_0^T \sum_{i=1}^n Y_i^\top Y_i(\dot{q}_i, \ddot{q}_i, \tau) d\tau \geq 0$, the only possibility is $\lambda_1 = 0$. This immediately implies that the set of $Y_i(\dot{q}_i, \ddot{q}_i, t)$'s are *not* collectively initially exciting, thus contradicting our premise. Similar arguments can be used to claim that $\int_0^T A(\dot{q}, \ddot{q}, t) dt > 0$.

From $\int_0^T C(t) dt > 0$ along with the facts that $k(t) > 0$, we can immediately conclude $\int_0^T \bar{Q}^2(t) dt \geq \lambda_c I_{mn}$ for some $\lambda_c > 0$. This immediately implies that $\int_0^{\max\{T, \bar{T}\}} \bar{Q}(t) dt > 0$. The union graph of $\mathcal{G}(t)$, denoted $\cup_{t \in [0, \max\{T, \bar{T}\}]} \mathcal{G}(t)$, is connected by the joint (δ, T) -connectedness assumption. The positive definiteness of the signless Laplacian for the union graph ($\int_0^{\max\{T, \bar{T}\}} \bar{Q}(t) dt$) and the joint connectivity of the union graph allows us to invoke Proposition II.2 and conclude that the union graph $\cup_{t \in [0, \max\{T, \bar{T}\}]} \mathcal{G}(t)$ is non-bipartite, i.e. $\mathcal{G}(t)$ is jointly non-bipartite over $[0, \max\{T, \bar{T}\}]$. \square

Remark V.1. We note here that [37] utilizes the non-bipartite graph structure to propose exponential bias estimators. The graph is, however, assumed to be constant. The necessary condition above allows the use of time-varying graphs that are only jointly non-bipartite (as opposed to at each time instant) and persists only for a finite time $[0, \max\{T, \bar{T}\}]$.

Based on Lemma V.1, assuming that we have a jointly non-bipartite union graph over $[0, \max\{T, \bar{T}\}]$, the primary purpose of $k(t)$ is to ensure that $\int_0^T A(\dot{q}, \ddot{q}, t) dt$ becomes positive definite. While there is no direct way to prescribe such a function for all possible initial conditions and system parameters, we introduce multiple frequency components through the time dependence in $k(t)$ to make the columns of $A(\dot{q}, \ddot{q}, t)$ linearly independent over sub-intervals.

VI. SIMULATION RESULTS

We now present simulation studies to verify Theorem IV.1 for a network of double integrators interacting via an undirected graph $\mathcal{G}(t)$ and bias corrupted measurements. We consider the translational dynamics of n identical quadrotors given by

$$\ddot{q}_i = \begin{bmatrix} 0 \\ 0 \\ -9.8 \end{bmatrix} + \mathbf{R}_i e_3 \frac{\tau_{1_i}}{M}, \quad \forall i = 1, 2, \dots, n \quad (24)$$

where the position vector is denoted by $q_i = (x_i, y_i, z_i)^\top \in \mathbb{R}^3$, $e_3 = (0, 0, 1)^\top$, $M \in \mathbb{R}^+$ is the mass of the quadrotor. $\mathbf{R}_i \in SO(3)$ is the 3×3 orthogonal rotation matrix from the quadrotor body frame to the inertial frame. The feedback $\tau_{1_i}(\cdot) \in \mathbb{R}$ is the sum of thrust forces from the individual motors in each quadrotor. Typical tracking control of the quadrotor consists of an inner loop attitude control [44] which modulates \mathbf{R}_i while the outer loop translation control is designed assuming full linear actuation in (24). Therefore, (24) can be treated as a double integrator model for our purposes by assuming a new control $u_i := \mathbf{R}_i e_3 \frac{\tau_{1_i}}{M}$. For these simulations we have considered $n = 5$.

Two adjacency matrices are used, one corresponding to a non-bipartite, connected graph(\mathcal{A}_b) and another corresponding to a bipartite, connected graph(\mathcal{A}_c).

$$\mathcal{A}_b = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad \mathcal{A}_c = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The graphs corresponding to \mathcal{A}_b and \mathcal{A}_c are shown in Figure 1 and Figure 2, respectively.

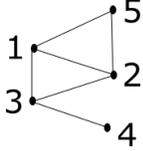


Fig. 1. Connected non-Bipartite graph

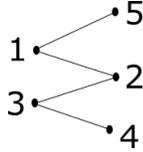


Fig. 2. Connected Bipartite graph

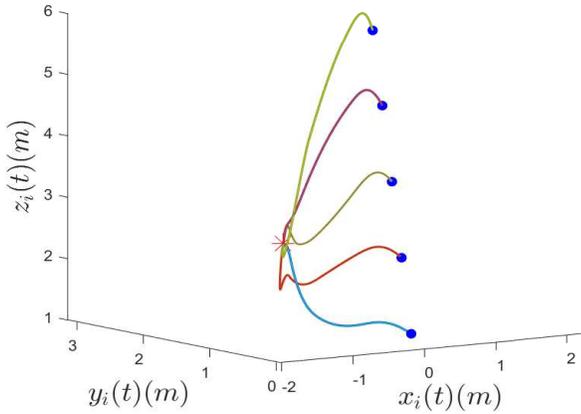


Fig. 3. $x_i(t)(m)$ vs $y_i(t)(m)$ vs $z_i(t)(m)$. The blue dots indicate start locations, and the red asterisk the terminal locations.

The actual adjacency matrix is obtained by cycling periodically (period $T = 8s$) between sub graphs of \mathcal{A}_b for the first $t = 8$ seconds and then between sub graphs of \mathcal{A}_c for the rest of the time. This allows for a jointly non-bipartite and connected graph in the initial phase of the simulations and a jointly bipartite, connected graph beyond.

The initial position, velocity, and the bias in relative measurement of the position for the i^{th} quadrotor are given by $[\frac{i\pi}{7}; \frac{i\pi}{5}; \frac{i\pi}{3}]$, $[0.1i - 0.7; -0.1i + 0.6; 0.1i + 0.7]$, $[\frac{i\pi}{12}; \frac{i\pi}{12}; \frac{i\pi}{12}]$ respectively for $i = 1, 2, \dots, 5$. $\hat{\theta}^i$ is initialized to the zero vector for $i = 1, 2, \dots, 5$. The gain constants σ , μ_F , μ_{IF} , λ , and β are chosen to be 0.2, 0.020, 15, 0.5, 0.5 respectively and $k(t) = 1 + 0.5\cos^2(t) + 0.5\sin^2(2t)$. The gain chosen helps introduce multiple frequency components through the time dependence in $k(t)$. The chosen $k(t)$ ensures that $\int_0^T A(\dot{q}, \ddot{q}, t) dt$ becomes positive definite, which guarantees collective initial excitation on the regressor Y_i 's.

Figure 3 is the phase-plane evolution of the three positions. As is evident, starting from different initial conditions, they converge to consensus. Similarly, all the three

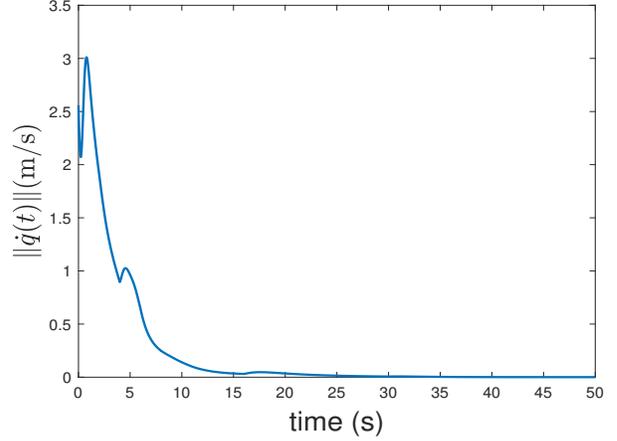


Fig. 4. $\|\dot{q}(t)\|(m/s)$ vs time (s).

velocities in Figure 4 and the bias estimation errors given by $\tilde{b} = [\tilde{b}^1, \tilde{b}^2, \tilde{b}^3, \tilde{b}^4, \tilde{b}^5]^T$ in Figure 5 converge to zero during the simulation horizon. The final plot, Figure 6 is for the verification of Lemma V.1, wherein we claim that the collective initial excitation of the regressors necessitates a jointly non-bipartite graph. Figure 6 plots the determinant of $\int_t^{t+T} Q(\tau) d\tau$ for all t , keeping $T = 4s$ as the cycling period. We see that the $\int_t^{t+T} Q(\tau) d\tau$ is positive definite over an initial period of time and beyond this is singular. This verifies, by Proposition II.2 that $\mathcal{G}(t)$ determined by $\mathcal{A}(t)$ is jointly non-Bipartite over a finite initial window.

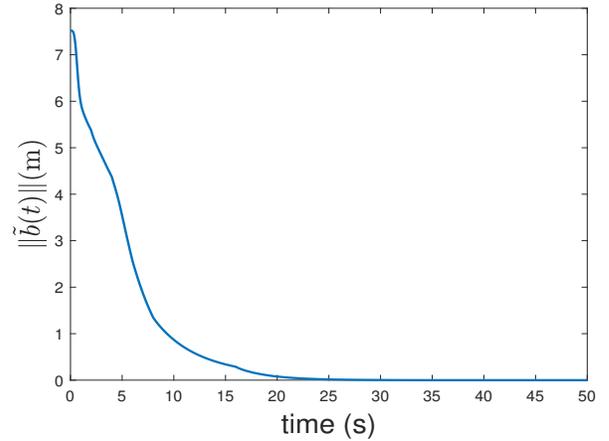


Fig. 5. $\|\tilde{b}(t)\|(m)$ vs time (s).

VII. CONCLUSION

In this article we propose a novel distributed adaptive controller to estimate bias in relative position measurements along with guaranteed exact consensus in a network of double-integrator systems. It is shown that joint (δ, T) -connectivity and joint non-Bipartite properties of the graph are necessary for bias estimation and consensus. In future work, we seek to explore more general measurement errors and nonlinear agent dynamics. We will also focus on consensus under erroneous

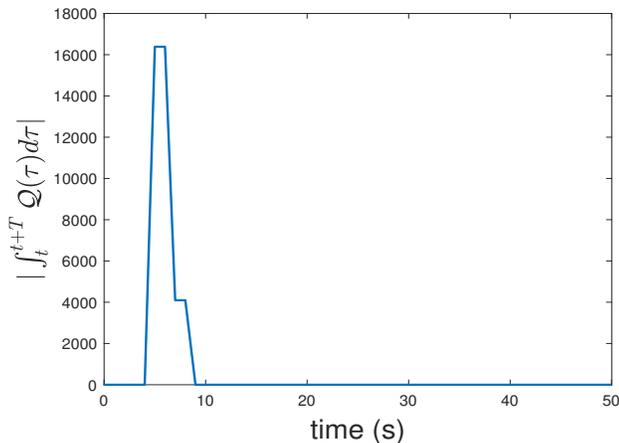


Fig. 6. Determinant of $\int_t^{t+T} Q(\tau) d\tau$ over time t , $T = 4s$.

relative measurements over directed and time varying communication graphs.

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